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THE MATHEMATICS TEACHER

Volume XLVI Number 2

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Pi and Probability

By Walter H. Carnahan

Purdue University, Lafayette, Indiana

One of the ancient weaknesses of men seems to be to take a chance and place a bet on its outcome. And one of his oldest scientific interests is that of the relation of diameter and circumference of a circle. In this brief article we shall call attention to the relation of this scientific interest to that of the observation of certain results of chance (but not the placing of bets).

Some two hundred years ago Buffon did an interesting experiment connecting π and probability. He tossed a needle onto a ruled surface, counted the tosses, and counted the number of times that the needle touched a line. Out of this experiment he found the value of π. This is a simple and interesting experience for high school pupils to repeat.

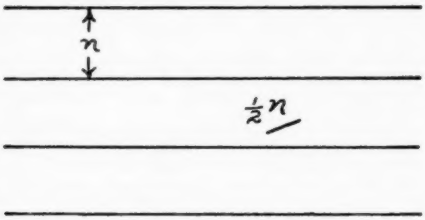

Fig. 1

Rule off a board or paper with equally spaced parallel lines n units apart. Cut a wire whose length is $n/2$ units. (This length is not necessarily $n/2$ but is suggested as a convenient one.) Now toss the wire at random onto the ruled surface. count tosses T and contacts C. After fifteen minutes or more, divide T by C The result is approximately equal to π.

The proof of this conclusion is not beyond the comprehension of a high school pupil. Suppose that the wire is bent into a circle; its circumference is $n/2$, and its radius is $n/4\pi$. Considered as a geometric line, the number of points on the needle is proportional to its length. (The philosophy underlying this statement might be debatable, of course.) Whether the needle is straight or bent, one point on it is just as likely to touch a line as is any other point. The shape of the needle will not affect the probability of any given point coming to rest on a line. Hence we can develop the discussion by assuming that the needle is bent into a circle. Since always two points on the circle will rest on the line if the line is in tangent or secant position, the probability of a one-point contact equals two times the probability of a secant relation.

A line will have a secant relation to the circle if the center of the circle is within radius distance of the line. The distance between two lines is n, and in the area between any two lines there are two areas $n/4\pi$ units wide in which the center of the circle could lie for a secant relation. Hence the probability of a secant relation is $n:n/2\pi=2\pi$. Therefore the probability of a one-point contact is π.

Now, the probability of contact when the needle is tossed is C/T. Hence $\pi=C/T$.

Another simple experiment for finding the value of π by using probability is

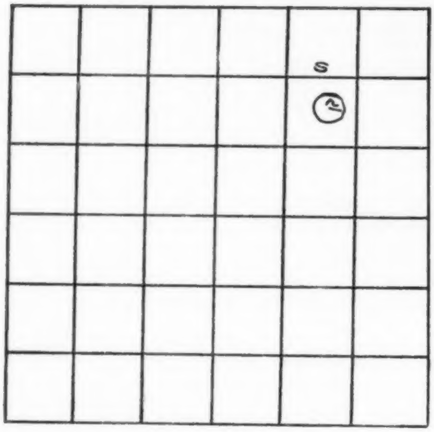

FIG. 2

tossing a coin onto a cross-ruled board. The distance between consecutive vertices of any square should be not less than the diameter of the coin; it may be greater than this. Toss the coin for fifteen minutes, count tosses T and count the number of times C the coin touches a vertex of a square. Multiply the area of a square by C, and divide this product by r^2T, r being the radius of the coin. This is approximately equal to π. It is convenient to take s the side of the square equal to $4r$. If this is done, then $\pi = 16C/T$.

The proof of this is simple. The area of the part of any square in which the center of the coin can fall for contact is the area of the coin (four quadrants). The total area in which the center of the coin can lie is the area of the square itself. Hence the formula as derived from consideration of the law of probability.

In rationalizing the result of the coin-tossing experiment our attention was on the center of the circle. The coin itself merely served to determine the size of undrawn circles on the board that contains the drawn squares. An alternative is to draw the circles and not draw the squares. The centers of the circles as shown in the figure are at the vertices of squares. The size of the circles or of the squares is not important so long as the circles do not overlap each other. For convenience let the radius r of each circle be 1 inch, and let the side s of each square be 4 inches. Toss darts at the board without aiming at any particular point on it. Count tosses T and the number of times C that a dart enters a circle. Divide $16C$ by T. This is approximately equal to π.

The reason for this is readily seen. Since the entire board is covered by squares (not drawn), the dart always enters a square. In every square there is a circle (in four quadrants) into which some of the darts will enter. The mathematical probability of a dart entering a circle is the ratio of the area of a circle to the area of a square, that is, $\pi r^2/s^2$. The experimental probability is C/T. Hence $\pi r^2/s^2 = C/T$, or $\pi = 16C/T$.

Any suitable device can be used to select the point on the board in the above experiment. If the board is level, a rolled marble would do, although with this device there is a difficulty of telling where the point of

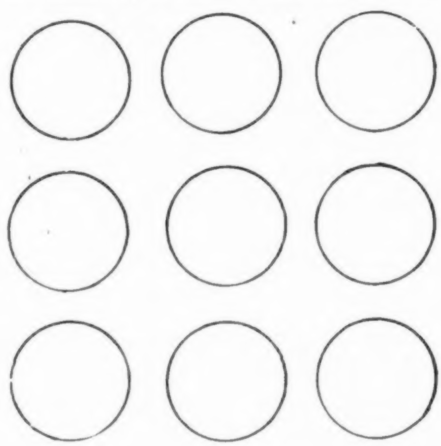

FIG. 3

contact of marble and board is located. A tossed disc with a hole through the center is very convenient. One can use a rifle or air pistol if one aims at the board in general and not at any particular point on it.

(*Continued on page 70*)

Trends in Geometry

By Jack D. Wilscn
San Francisco State College, San Francisco, California

The reason given for teaching plane geometry in high school has usually been its value as a course in critical thinking. This is a most desirable objective and, if established as pedagogically feasible, makes a strong case for the subject. In the twenties many writers stated that nothing else approached geometry in this phase of the work, that nowhere else, except in pure logic, did the pupil marshal his facts in such strict order and present them with such precision of statement. Further, geometry was thought to be superior to pure logic because it had in it figures that illustrated the things that were said and done. "If geometry is not taught to give a training in deductive logic, in the power to prove one's own statements, or if it does not do both, then it is not worth teaching." (8, pp. 18–19)

Nevertheless, although geometry was defended as a course in reasoning, it was generally recognized that in practice it often degenerated into sterile memory work. Barber described the classroom situation as follows: (1, p. 126)

> Teacher, text and pupil are apt to pursue each other around in a vicious circle. The authors think what we want is a page easy to fix in mind. The pupil, seeing the page, thinks that we want him to learn it. When he has done so and has repeated glibly, we are prone to put the stamp of approval upon his memory work.

This state of affairs contributed to the widespread attacks on high school mathematics. Mathematics educators began to view with alarm the apparent trend to eliminate geometry (and algebra) from the programs of most high school pupils. David E. Smith said: (10, p. 21)

> It would be disastrous to sacrifice the courses in algebra and geometry that we now have, since we have nothing worthy of the student's vigorous mental attack that can take their place.

Many authorities believed that geometry itself was not at fault but rather the methods of teaching. They suggested that book propositions should receive less attention, that emphasis should be placed, instead, on original exercises and the development of analysis.

During the thirties the values claimed for the original exercise began to be questioned. Little concrete evidence of transfer of reasoning processes from the original exercises of geometry to real life situations was forthcoming. However, mathematics educators were unwilling to concede that geometry could not be made a vehicle for teaching clear thinking. Fawcett and many others proposed a further revision of methods and content with greater emphasis on the nature of proof. Not only should students understand the nature of proof but their way of life should show that they understand it. It was suggested that, while the total educational experience of the student should contribute to the development of ability to reason, there were possibilities in demonstrative geometry which no other subject offered because in this area the concepts considered and the ideas studied were devoid of strong emotional content. But many mathematics educators now believed that: (3, pp. 12–13)

> If the kind of thinking which is to result from an understanding of the nature of proof is to be used in nonmathematical situations such situations must be considered during the learning process.

The results of several experiments were reported in support of this point of view, of which the better known are the studies by Fawcett and by Ulmer. Fawcett used interesting nonmathematical situations to introduce pupils to the importance of definition and to the fact that conclu-

sions depend on assumptions. Space study was not neglected. Each pupil developed his own text, selected his own undefined terms, constructed his own definitions, decided on the assumptions as a product of his own thinking, and discovered his own proofs. Many applications were made to nonmathematical situations which might be encountered in life.

The amount of literature devoted to "reasoning in life situations" in geometry decreased substantially during the forties. Voices began to be raised in opposition to this approach. Mallory and Fehr said: (6, p. 292)

> Another movement that has weakened the mathematics program is the introduction of an excessive amount of *Reasoning in Life Situations* into the subject of geometry. In many cases this has resulted in befuddled thinking and a lack of knowledge of plane geometry. . . . The primary purposes of geometry are to teach the facts of space and the nature of mathematical thought.

This quotation brings forward another recognized objective of plane geometry, namely, to give information concerning the facts and principles of space, including information which serves as a background for the appreciation of the mechanical and industrial life of today. This objective is not new. It was with us in the twenties and the thirties but usually occupied a secondary position. In the forties a number of articles hinted that it might be coming forward to challenge "critical thinking" as a major goal. Van Waynen said: (13, p. 64)

> The title "geometry" suggests the science of earth measure, while its place in the mathematics curriculum implies that the emphasis is on logic. Which side is right will depend on the results obtained. Teachers of geometry are on the verge of making a momentous decision.

Kinney implied that such a choice might have to be made when he wrote: (5, p. 4)

> Is it geometry for the purpose of developing critical thinking, the nature of proof, ability to analyze propaganda, and so on? Or is the nature of proof to be incidental to the study of geometric relationships, constructions, or applications in technical, industrial, or scientific problems? Attempts to do both in the same course have failed in the past.

The nineteenth yearbook of the National Council is devoted to the history and classroom use of surveying instruments, and may have some influence on classroom practice.

Courses of study did give considerable attention both to critical thinking and to the experimental study of space relationships or "earth measure" during the forties. One state course (11) suggested that teachers follow the life situations nature of proof method of Fawcett and Ulmer but at the same time asked that extra effort be made to give pupils concrete problems and actual use of instruments, such as simple student-made transit, clinometer, and astrolabe, and suggested a wide area of applications—gears; aviation; perspective mapping; use of pantograph, plane table, cross staff; scale drawings; and models. This double emphasis poses a difficult problem for the geometry teacher and may cause some frustration if neither objective is attained.

Recently expressed opinions of research mathematicians make clear their position on these two objectives. Eric T. Bell, emphasizing that the essence of mathematics is deductive reasoning from explicitly stated assumptions, considers (2, p. 260) that geometric exercises in cutting, weighing, and measuring are merely scissor-and-balance gymnastics, which, as an introduction to plane geometry, are "silly, incompetent, immaterial, and irrelevant." George Polya reminds us (7, p. 3) that "mathematics (especially geometry) is the only high school subject . . . in which the student can solve problems on a scientific level." When a pupil discovers for himself the proof of a simple theorem on the level of Euclid, he is doing something not possible in any other high school subject.

These views are undeniably sound but in school practice may add up to a reiteration of the old algebra-geometry theme, a theme now viewed with considerable skepticism. Says Donald Kauffman, former president of the California Mathematics Council: (4, p. 4)

I am filled with the uneasy feeling that I am participating, year after year, in the production of yet another generation of adults to whom algebra and geometry will be recalled only as hurdles crossed in getting out of high school or into college, adults whose failure to find useful in life beyond the classroom walls what they toiled to learn in two valuable and impressionable high school years constitutes our severest indictment.

Two of the most significant developments at the present time are the experimental curricula being tried out in New York and in southern California. The State of New York tenth year mathematics curriculum, revised on the basis of a three year study carried on in twenty-five schools throughout the state, may be described as general mathematics inasmuch as algebra, arithmetic, trigonometry, and coordinate geometry are included, but it is clear that the development of logical thinking through demonstrative geometry is the primary aim: (12, p. 5)

The concepts of the nature of proof as well as the logical structure exhibited in a chain of propositions are essential objectives in the teaching of formal geometry.

This is in line with tradition. What is new is all-out support for extensive use of the methods of geometry to assist critical thinking in nonmathematical settings: (12, p. 29)

Without a conscious, continuous and persistent effort on the part of the teacher and the student to correlate the thinking in geometry with that in non-mathematical fields, very little, if any, improvement in the ability to think soundly and critically may be expected.

New York has made the "momentous decision."

The Los Angeles program (9) embodies the complete reorganization of high school mathematics for college preparatory students other than mathematics, engineering, and science majors. Los Angeles is well along in the development of a four-semester integrated course which includes the most useful parts of arithmetic, algebra, geometry, trigonometry, and consumer mathematics. At various points throughout the four semesters are listed "earth measurement" topics; scale drawings; indirect measurement; applications to industry; surveying; navigation; use of plane table; etc. In the fourth semester, ten weeks are given to demonstrative geometry in the traditional pattern. The outline does not emphasize application of the methods of geometry in nonmathematical settings, but, with reorganization on such a grand scale, probably there has not been time to develop this area completely nor to indicate actual classroom activities.

In our story of trends in plane geometry, we have noted two major objectives, both extremely worthwhile. Critical thinking is necessary if each of us is to participate in the democratic way of life. Knowledge of the facts of geometry and their application to life situations is essential in an industrialized society. The possibility of achieving both objectives at one and the same time has been questioned. Perhaps this indicates our schools should offer alternative senior high school plane geometry courses: (1) a "nature of proof" course; and (2) a "science of earth measurement" course organized around eight or ten large problem units in such a way that the facts and principles of the subject are mastered through extensive use of instruments in classroom, in laboratory, and in field work. However, a four-semester course, as in Los Angeles, may offer the best opportunity to achieve both major objectives in a systematic manner.

This problem should be given careful consideration during the pre-service education program of the future mathematics teacher for it is the classroom teacher who determines the kind of geometry taught in the schools of the nation.

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Pi and Probability

(Continued from page 66)

Small circles on a board placed at a great distance will help. (See Fig. 3)

The preceding experiment can be varied by drawing a set of ellipses on rectangles rather than circles on squares. We shall not go through the details. We might repeat the familiar textbook statement that "This is left as an exercise for the student." If a and b are the semiaxes of the ellipses, and if $3a$ and $3b$ are sides of each rectangle, then $\pi = 9C/T$. (The area of an ellipse is πab.)

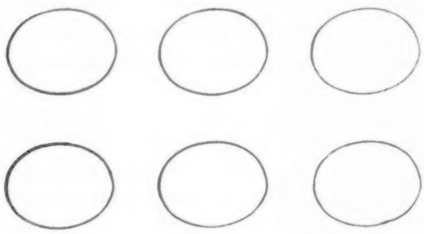

FIG. 4

There are numerous possible variations of the devices suggested: One square in one circle; one circle in one square; one ellipse on one rectangle. One can even cut up the squares or circles and scatter the pieces so long as they do not overlap. Or one can cut the figures and arrange the pieces in patterns. The ratio of the areas is the essential consideration.

Valentine

By KATHARINE O'BRIEN
Deering High School, Portland, Maine

You disintegrate my differential,
You dislocate my focus.
My pulse goes up like an exponential
Whenever you cross my locus.
Without you sets are null and void,
So won't you be my cardioid?

Hyperbolic Functions

By Aaron Bakst
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The development of *hyperbolic functions* in the traditional trigonometry courses (if this is ever reached during a one-semester instruction) is usually confined to purely algebraic methods. However effective the latter procedures may be, it is doubtful that a student realizes the import of the properties of hyperbolic functions. The student is never offered the opportunity to realize the fact that, essentially, the properties of hyperbolic functions are analogous to the properties of circular functions. It is possible, however, to develop the properties of hyperbolic functions in a manner which is analogous to the processes which are employed in the development of circular functions. Thus, it is proposed to examine and to develop hyperbolic functions by means of a geometric approach.

We shall consider the equilateral hyperbola

$$xy=a \tag{1}$$

whose properties will be examined presently. We shall formulate these properties in terms of certain propositions.

Proposition 1. If the lengths of the straight line segments on the coordinate axis OX which are bounded by the origin and the projections B_1, B_2, B_3, and B_4 of the points A_1, A_2, A_3, and A_4 respectively on the equilateral hyperbola $xy=a$ are proportional, then the areas of the rectangular trapezoids $A_1A_2B_2B_1$ and $A_3A_4B_4B_3$ are equal.

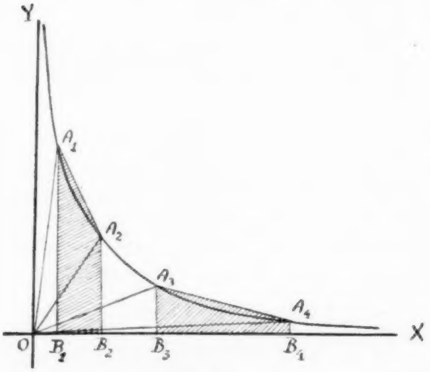


Fig. 1.

The areas of the trapezoids $A_1A_2B_2B_1$ and $A_3A_4B_4B_3$ (Fig. 1) are

$$\begin{aligned} S_1&=\tfrac{1}{2}(x_2-x_1)(y_1+y_2) \quad \text{and} \\ S_2&=\tfrac{1}{2}(x_4-x_3)(y_3+y_4) \end{aligned} \tag{2}$$

respectively.

Replacing the y's in the equation (2) by their corresponding expressions in terms of the x's with their appropriate subscripts (see equation (1)) and performing the algebraic simplifications, we obtain

$$S_1=a\left(\frac{x_2}{x_1}-\frac{x_1}{x_2}\right) \quad \text{and}$$

$$S_2=a\left(\frac{x_4}{x_3}-\frac{x_3}{x_4}\right). \tag{3}$$

Since

$$x_1:x_2=x_3:x_4, \quad S_1=S_2.$$

The converse of Proposition 1 may be established by equating the relations (3). Let us denote $x_2:x_1$ by m and $x_4:x_3$ by n. We then have

$$m-\frac{1}{m}=n-\frac{1}{n}\cdot$$

We then obtain $m^2n-n=n^2m-m$ or $m^2n+m=n^2m+n$. From this we obtain $m=n$ or $x_1:x_2=x_3:x_4$.

Proposition 2. If the lengths of the straight line segments on the coordinate axis OX which are bounded by the origin and the projections B_1, B_2, B_3, and B_4 of the points A_1, A_2, A_3, and A_4 respectively on the equilateral hyperbola $xy=a$ are proportional, then the areas of the curvilinear trapezoids $A_1A_2B_2B_1$ and $A_3A_4B_4B_3$ are equal.

Divide each of the straight line segments A_1A_2 and A_3A_4 (Fig. 1) into n equal parts (n an integer) and mark off on the hyperbola the corresponding ordinates of the abscissas of the division points thus obtained. Join these points on the hyperbola with their corresponding abscissas by straight line segments. Furthermore, join the points on the hyperbola thus obtained by straight line segments. We thus obtain two polygonal areas, each of them consisting of n trapezoids. The two polygonal areas are equal according to Proposition 1. This may be noted from the fact that the first small trapezoid in $A_1A_2B_2B_1$ has an area which is equal to the area of the first trapezoid in $A_3A_4B_4B_3$. The same may be observed for the pair of the second trapezoids in the two polygons, and so on. Allowing n to approach infinity and considering the limiting case, we may obtain the limits of the two polygonal areas $A_1A_2B_2B_1$ and $A_3A_4B_4B_3$. The limits of these two polygonal areas are equal.[1]

It may be noted that the converse of Proposition 2 is also true. The proof of this converse follows the procedures used in the proofs of the above two propositions.

Proposition 3. If the lengths of the straight line segments on the coordinate axis OX which are bounded by the origin and projections B_1, B_2, B_3, and B_4 of the points A_1, A_2, A_3, and A_4 respectively on the equilateral hyperbola $xy=a$ are proportional, then the areas of the sectors OA_1A_2 and OA_3A_4 are equal (Fig. 1).

Let us denote the area of the curvilinear trapezoid by $(A_1A_2B_2B_1)$. Then Area of sector $OA_1A_2=$ Area of $(A_1A_2B_2B_1)+$ Area of $\triangle OA_1B_1-$ Area of $\triangle OA_2B_2$.

By virtue of the equation of the equilateral hyperbola $xy=a$,

Area of $\triangle OA_1B_1=$ Area of $\triangle OA_2B_2$.

Thus,

Area of sector $OA_1A_2=$ Area of $(A_1A_2B_2B_1)$

[1] It should be understood that this is only an intuitive approach to the proof of this proposition. A rigorous proof of this proposition may be obtained by means of integral calculus.

Similarly,

Area of sector $OA_3A_4=$ Area of $(A_3A_4B_4B_3)$.

Then, by virtue of Proposition 2,

Area of sector OA_1A_2

$=$ Area of sector OA_3A_4.

It should be noted that the converse of this proposition is also true. The proof of the converse may be obtained by establishing the fact that the areas of the triangles OA_1B_1 and OA_2B_2 are equal. From this will follow that the areas of the curvilinear trapezoids $A_1A_2B_2B_1$ and $A_3A_4B_4B_3$ are equal. Then this converse follows from the converse of Proposition 2.

Proposition 4. If the point A is the midpoint of the chord A_1A_2 of the equilateral hyperbola $xy=a$, then the straight lines which are drawn through the points A_1 and A_2 parallel to the asymptotes of the equilateral hyperbola intersect on the straight line OA (Fig. 2).

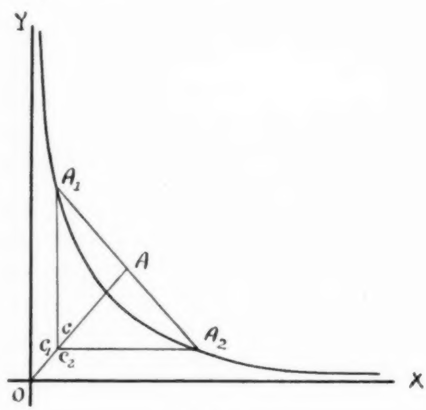


Fig. 2

Since A_1C_1 is parallel to OY,

$$\frac{OA}{OC_1}=\frac{x_1+x_2}{2x_1}.$$

Since A_2C_2 is parallel to OX,

$$\frac{OA}{OC_2}=\frac{y_1+y_2}{2y}.$$

The right members of these two equations are equal by virtue of the equation of the equilateral hyperbola $xy=a$. Hence

$$OA:OC_1=OA:OC_2,$$

and the points C_1 and C_2 are coincident.

The converse of this proposition is also true.

PROPOSITION 5. If the point A is the midpoint of the chord A_1A_2 of the equilateral hyperbola $xy=a$, then the straight lines OA and A_1A_2 make equal angles with the asymptotes of the equilateral hyperbola (Fig. 3).

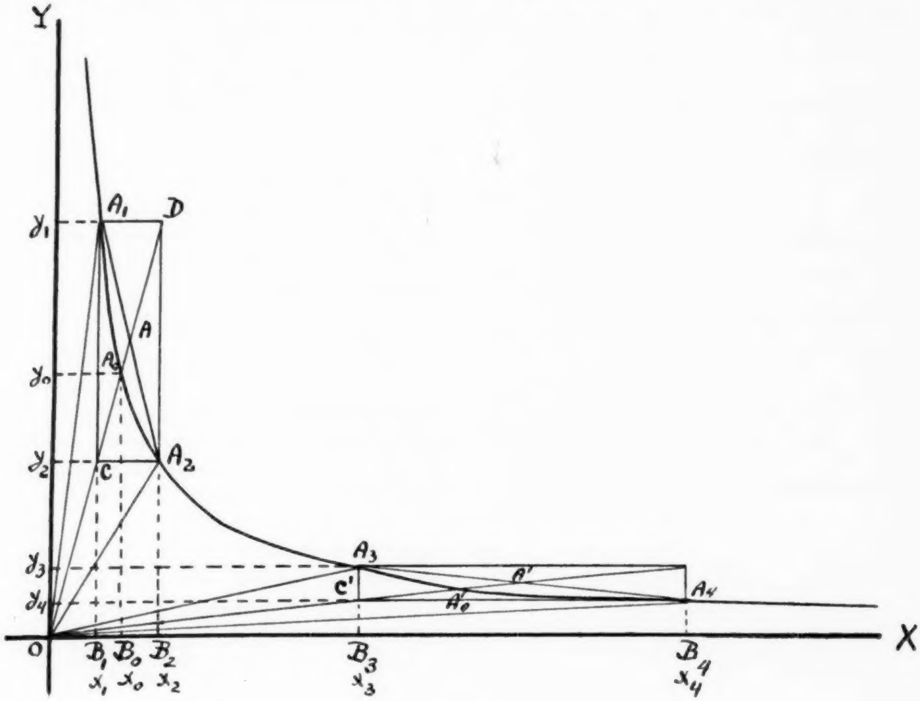


FIG. 3

According to Proposition 4 we have a right triangle A_1CA_2 in which the straight line AC is the median. In other words, the triangles A_1CA and ACA_2 are both isosceles. Then $\angle AA_1C=\angle ACA_1$ and $\angle ACA_2=\angle AA_2C$.

Proposition 5 is very important because from this theorem it follows that the direction of the diameter which is conjugate with its chord (OA and A_1A_2 respectively) depends on the direction of the chord A_1A_2. Hereafter we shall refer to the directions of OA and A_1A_2 as *conjugate directions.*

PROPOSITION 6. If A is the midpoint of the chord A_1A_2 of the equilateral hyperbola $xy=a$, then the straight line OA intersects the hyperbola in the point A_0 whose coordinates (x_0, y_0) are the mean proportionals of the respective coordinates of the points A_1 and A_2 (Fig. 3).

The equation of the equilateral hyperbola leads to

$$x_0y_0=x_1y_1=x_2y_2.$$

From the similarity of the triangles ODB_2 and OA_0B_0 we have

$$x_0:y_0=x_2:y_1.$$

Multiplying this equation and the equation $x_0y_0=x_1y_1$ term by term, we obtain

$$x_0^2=x_1x_2.$$

Multiplying term by term the equations $x_0y_0 = x_2y_2$ and $y_0 : x_0 = y_1 : x_2$ we obtain

$$y_0^2 = y_1y_2.$$

As a corollary, we find that the straight line OA_0 bisects the area of the sector OA_1A_2. This follows from the results of Proposition 3 and from the relation

$$x_1 : x_0 = x_0 : x_2.$$

PROPOSITION 7. If Area of sector OA_1A_2 = Area of sector OA_3A_4 (Fig. 3), and A and A' are the midpoints of the chords A_1A_2 and A_3A_4 of the equilateral hyperbola $xy = a$, then we have the proportion

$$OA_0 : OA : A_1A_2 = OA_0' : OA' : A_3A_4.$$

Since the areas of the sectors are equal, we have, by virtue of the converse of Proposition 3,

$$x_1 : x_2 = x_3 : x_4.$$

According to Proposition 4

$$\frac{OA}{OC} = \frac{x_1 + x_2}{2x_1} \quad \text{and} \quad \frac{OA'}{OC'} = \frac{x_3 + x_4}{2x_3}.$$

Then

$$\frac{OA}{OC} = \frac{OA'}{OC'} \quad \text{or} \quad \frac{OA}{OA'} = \frac{OC}{OC'}. \tag{4}$$

Furthermore,

$$\frac{OC}{OD} = \frac{OC'}{OD'}. \tag{5}$$

According to Proposition 6,

$$OA_0^2 = OA \cdot OC$$

and

$$OA_0'^2 = OA' \cdot OC'.$$

From this it follows that

$$\frac{OA_0}{OA_0'} = \frac{OA}{OA'} = \frac{OC}{OC'}. \tag{6}$$

Since

$$\frac{OD}{OC} = \frac{OC + A_1A_2}{OC} = 1 + \frac{A_1A_2}{OC}$$

and

$$\frac{OD'}{OC'} = \frac{OC' + A_3A_4}{OC'} = 1 + \frac{A_3A_4}{OC'}.$$

Then, by virtue of (5),

$$\frac{A_1A_2}{A_3A_4} = \frac{OC}{OC'}. \tag{7}$$

The proportions (4), (6), and (7), when combined, establish the proof of Proposition 7, that is,

$$OA_0 : OA : A_1A_2 = OA_0' : OA' : A_3A_4.$$

From Proposition 7 we obtain another result. The proportions

$$\frac{AA_1}{OA_0} = \frac{A'A_3}{OA_0'} \quad \text{and} \quad \frac{OA}{OA_0} = \frac{OA'}{OA_0'}$$

are functions of the areas of the sectors OA_1A_2 and OA_3A_4 (these areas being equal) for any given equilateral hyperbola. Let us denote the areas of these sectors by T. We may then denote these functional relations as follows:

$$\frac{AA_1}{OA_0} = \sinh T \quad \text{and} \quad \frac{OA}{OA_0} = \cosh T.$$

The construction of the hyperbolic sine and hyperbolic cosine are carried out as follows. According to the corollary of Proposition 6, the straight line OA_0 bisects the area of the sector OA_1A_2. We construct the sector $OA_1A_0 = 1/2\ T$. Through the point A_1 we draw a straight line A_1A which should be *conjugate* with the straight line OA_0.[2]

According to Proposition 6 (Fig. 3), $x_0^2 = x_1x_2$.

Then,

[2] The definition of the hyperbolic sine and hyperbolic cosine given above is analogous in many respects with the definition and development given by E. W. Hobson in *A Treatise on Plane Trigonometry*, Cambridge: at the University Press, 1921, pp. 329–330. However, the departure here from Hobson's treatment (he draws an analogy with the properties of the circle and the area of a circular sector) consists in the consistent employment of the properties of the hyperbola.

$$\frac{OC}{OA_0}=\frac{x_1}{x_0} \quad \text{and} \quad \frac{OA_0}{OD}=\frac{x_0}{x_2},$$

or,

$$\frac{OC}{OA_0}=\frac{OA_0}{OD}\cdot$$

Then,

$$\begin{aligned} OA_0^2 &= OC\cdot OD=(OA-AC)(OA+AD)\\ &=(OA-AD)(OA+AD)\\ &=OA^2-AD^2,\\ OA_0^2 &= OA^2+AA_1^2. \end{aligned}$$

Hence,

$$\frac{OA^2}{OA_0^2}-\frac{AA_1^2}{OA_0^2}=1,$$

or,

$$\cosh^2 T-\sinh^2 T=1.$$

The derivation of the expression for the hyperbolic functions of the sum of two angles is analogous to the derivation of the expressions for circular functions. We will demonstrate the complete analogy by using two diagrams with identical letterings (Figs. 4*a* and 4*b*).

Let Area of sector $OAB=1/2\ a$ and Area of sector $OBC=1/2\ b$ (Fig. 4*b*). Then, Area of sector $OAC=1/2(a+b)$.

In order to construct the hyperbolic functions we proceed as follows. Draw the straight line BH making the same angles with the asymptotes of the hyperbola as the straight line OA. In other words, BH and OA will have conjugate directions. Then draw CG parallel to BH. In a similar manner, draw CE so that it will be conjugate with OB. Then,

$$\sinh a=\frac{BH}{OA}, \quad \sinh b=\frac{CE}{OB}, \quad \text{and}$$

$$\sinh(a+b)=\frac{CG}{OA}\cdot$$

$$\cosh a=\frac{OH}{OA}, \quad \cosh b=\frac{OE}{OB}, \quad \text{and}$$

$$\cosh(a+b)=\frac{OG}{OA}\cdot$$

The expressions for the circular functions may be obtained from Figure 4*a*.

Draw in Figures 4*a* and 4*b* straight lines EF, BH, and CG parallel to each other. Also, draw ED parallel to OA. Then,

$$\sinh(a+b)=\frac{CG}{OA}=\frac{CD}{OA}+\frac{DG}{OA}=\frac{EF}{OA}+\frac{DC}{OA}\cdot$$

From the similarity of the triangles OEF and OBH we have:

$$EF:BH=OE:OB.$$

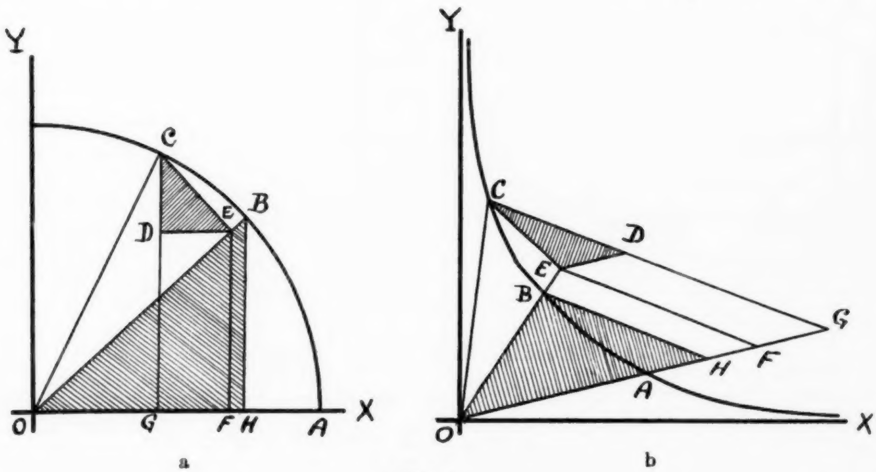


Fig. 4

Then,

$$\frac{EF}{OA}=\frac{BH}{OA}\cdot\frac{OE}{OB}=\sinh a\cdot\cosh b.$$

From the similarity of the triangles DEC and OBH we have:

$$DC:OH=EC:OB.$$

Then,

$$\frac{DC}{OA}=\frac{OH}{OA}\cdot\frac{EC}{OB}=\cosh a\cdot\sinh b.$$

Finally,

$$\sinh(a+b)=\sinh a\cdot\cosh b+\cosh a\cdot\sinh b.$$

Similarly,

$$\cosh(a+b)=\frac{OG}{OA}=\frac{OF}{OA}+\frac{FG}{OA}\cdot$$

Using the similarity properties of the above two pairs of triangles, we have:

$$\frac{OF}{OA}=\frac{OH}{OA}\cdot\frac{OE}{OB}=\cosh a\cdot\cosh b,$$

and

$$\frac{FG}{OA}=\frac{ED}{OA}=\frac{BH}{OA}\cdot\frac{EC}{OB}=\sinh a\cdot\sinh b.$$

Finally,

$$\cosh(a+b)=\cosh a\cdot\cosh b+\sinh a\cdot\sinh b.$$

The process of the derivation of the expressions for hyperbolic functions employs the property of *conjugate directions* with respect to a hyperbola, while the process of the derivation of the expressions for circular functions employs the property of *perpendicularity*. Thus, *perpendicularity* represents *conjugate directions* with respect to a circle.

The function

$$f(a)=\cosh a+\sinh a$$

satisfies the functional equation

$$f(a)\cdot f(b)=f(a+b).$$

We have:

$$\begin{aligned} f(a)\cdot f(b) &\\ &=(\cosh a+\sinh a)\cdot(\cosh b+\sinh b)\\ &=\cosh(a+b)+\sinh(a+b). \end{aligned}$$

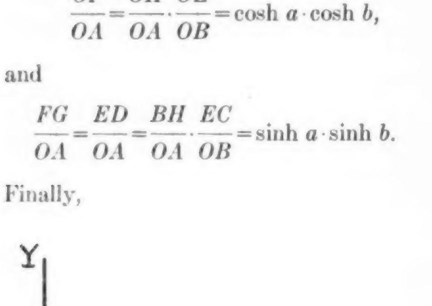
Thus,

$$f(a)\cdot f(b)=f(a+b).$$

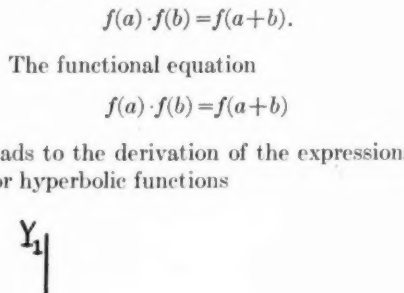
The functional equation

$$f(a)\cdot f(b)=f(a+b)$$

leads to the derivation of the expressions for hyperbolic functions

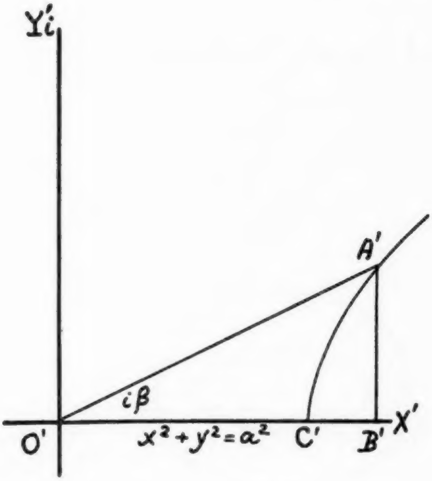


a

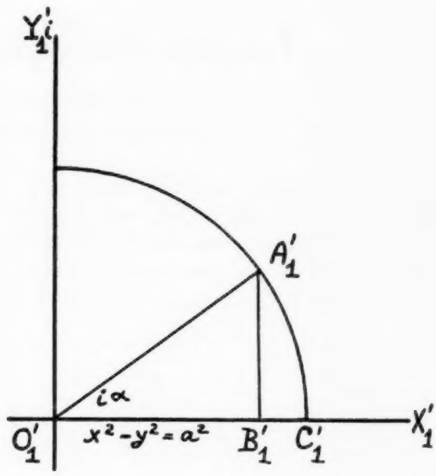


b

Fig. 5



a



b

Fig 6

$$\sinh a = \tfrac{1}{2}(e^a - e^{-a}) \text{ and } \cosh a = \tfrac{1}{2}(e^a + e^{-a}),$$

where e is the base of the natural logarithms.

The structural relationship between hyperbolic functions and circular functions manifests a striking similarity if these two types of functions are considered in the real plane and in the complex plane.

In the real plane the coordinates of a point in the plane are represented by real numbers. In the complex plane, the abscissas are represented by real numbers, while the ordinates are represented by imaginary numbers. Thus, in the complex plane the coordinate axis OY is known as the *imaginary axis*.

In the real plane, the equation of a circle (with the center at the origin) is $x^2+y^2=a^2$, and the equation of a hyperbola (associated with the same circle) is $x^2-y^2=a^2$. The graphs of these two curves are represented in Figures 5*a* and 5*b*.

The equation of the circle in the complex plane is $x^2-y^2=a^2$, while the equation of the hyperbola in the complex plane is $x^2+y^2=a^2$. The graphs of these two curves are represented in Figures 6*a* and 6*b*.

For the sake of simplicity we shall assume that $a=1$.

We have then the following expressions:

$\sin \alpha = AB, \quad \cos \alpha = OB \quad$ (Fig. 5*a*)

$\sin i\beta = A'B', \quad \cos i\beta = O'B'$ (Fig. 6*a*)

$\sinh \beta = A_1B_1, \quad \cosh \beta = O_1B_1$ (Fig. 5*b*)

$\sinh i\alpha = A_1'B_1',$

$\cosh i\alpha = O_1'B_1'$ (Fig. 6*b*)

$A_1'B_1' = i\cdot AB$. Therefore, $\sinh i\alpha = i \sin \alpha$.

$O_1'B_1' = OB$. Therefore, $\cosh i\alpha = \cos \alpha$.

$A'B' = i\cdot A_1B_1$. Therefore, $\sin i\beta = i \sinh \beta$.

$O'B' = O_1B_1$. Therefore, $\cos i\beta = \cosh \beta$.

HAVE YOUR STUDENTS SEEN?

In *Holiday*,
"Proof by Nine" by Ruth McKenney in the October 1952 issue
In *Scientific American*
"Running Records" by M. H. Lietzke in the August 1952 issue
"Is There an Infinity" by Hans Hahn in the November 1952 issue
"Crystals and the Future of Physics" by Phillipe Le Corbeiller in the January 1953 issue

Tangent Circles and Conic Sections

By WILLIAM GILBERT MILLER
Clemson College, Clemson, South Carolina

A CONIC SECTION may be defined as the locus of the center of a circle which is tangent to two given circles. In fact, for every combination of two given circles two conic sections will be defined.

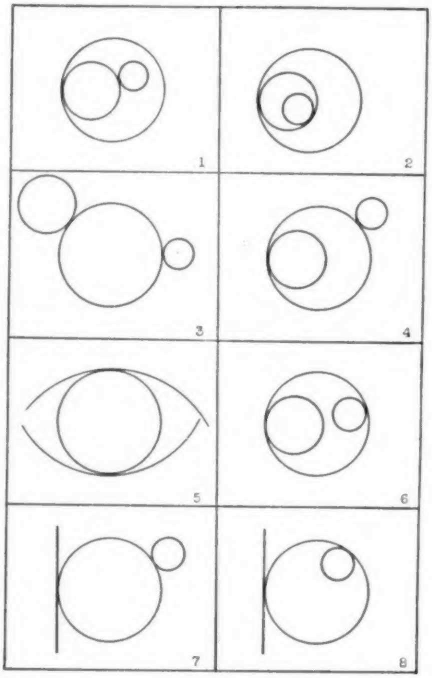


FIG. 1

Figure 1 illustrates the eight combinations of tangency pertinent to this discussion.

Let R designate the radius of the given circle with center at C, and r designate the radius of the given circle with center at c. No generality is lost by taking $R \geqq r$. Let ρ_n denote the radii of the generating circles with centers E_n (describing an ellipse), H_n (describing a hyperbola) and P_n (describing a parabola). The subscript indicates which of the eight conditions of tangency of Figure 1 apply.

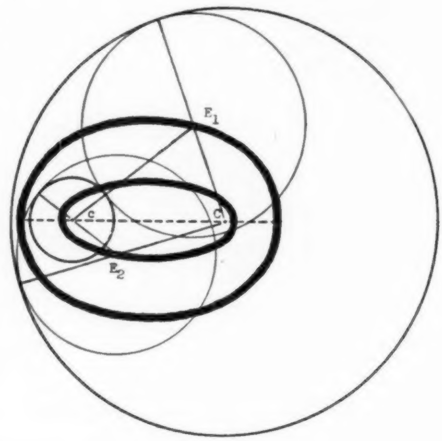


FIG. 2

(a) Case 1. Where $(R-r) \geqq Cc \geqq 0$.

From Figure 2:

$$\begin{array}{rcl} CE_1 = R-\rho_1 & \qquad & CE_2 = R-\rho_2 \\ cE_1 = r+\rho_1 & & cE_2 = \rho_2 - r \\ \hline CE_1 + cE_1 = R+r & & CE_2 + cE_2 = R-r \end{array}$$

Thus E_1 generates an ellipse with foci at C and c, and with major axis equal to $(R+r)$; E_2 generates an ellipse with foci at C and c, and with major axis equal to $(R-r)$.

(b) Case 2. Where $Cc \geqq (R+r) \geqq (R-r) \geqq 0$.

From Figure 3:

$$\begin{array}{rcl} CH_3 = R+\rho_3 & \qquad & CH_6 = \rho_6 - r \\ cH_3 = r+\rho_3 & & cH_6 = \rho_6 - R \\ \hline CH_3 - cH_3 = R-r & & cH_6 - CH_6 = R-r \end{array}$$

Thus H_3 and H_6 generate the two branches of the hyperbola with foci at C

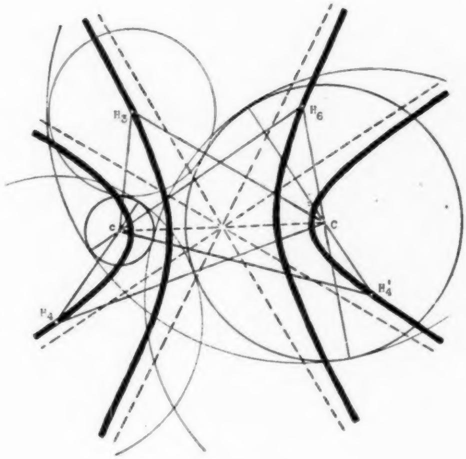


FIG. 3

and c, and with transverse axis equal to $(R-r)$.

Also from Figure 3:

$$\begin{array}{ll} CH_4 = R+\rho_4 & cH_4' = r+\rho_4' \\ cH_4 = \rho_4 - r & CH_4' = \rho_4' - R \\ \hline CH_4 - cH_4 = R+r & cH_4' - CH_4' = R+r \end{array}$$

Thus H_4 and H_4' generate the two branches of the hyperbola with foci at C and c, and with transverse axis equal to $(R+r)$.

(c) Case 3. Where $(R+r) \geqq Cc \geqq (R-r) \geqq 0$.

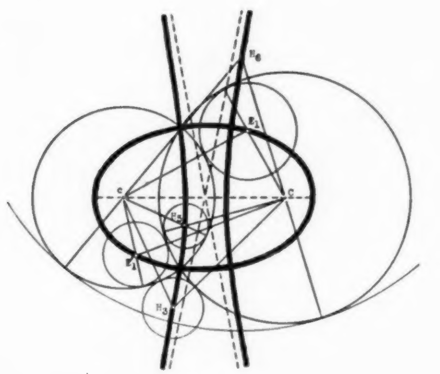


FIG. 4

From Figure 4:

$$\begin{array}{ll} CE_1 = R-\rho_1 & CE_1' = R+\rho_1' \\ cE_1 = r+\rho_1 & cE_1' = r-\rho_1' \\ \hline CE_1 + cE_1 = R+r & CE_1' + cE_1' = R+r \end{array}$$

Thus E_1 and E_1' generate the ellipse with foci at C and c, and with major axis equal to $(R+r)$.

Also from Figure 4.

$$\begin{array}{ll} CH_3 = R+\rho_3 & CH_5 = R-\rho_5 \\ cH_3 = r+\rho_3 & cH_5 = r-\rho_5 \\ \hline CH_3 - cH_3 = R-r & CH_5 - cH_5 = R-r \end{array}$$

$$\begin{array}{l} cH_6 = \rho_6 - r \\ CH_6 = \rho_6 - R \\ \hline cH_6 - CH_6 = R-r \end{array}$$

Thus H_3, H_5 and H_6 generate the hyperbola with foci at C and c, and with transverse axis equal to $(R-r)$.

In the two following cases let R approach infinity and let AB represent an arc of the circle with center at C, AB lying in the vicinity of c. Let N designate the projection of c on AB.

(d) Case 4. Where $cN \geqq r$.

From Figure 5:

$$cP_7 = r+\rho_7 = DP_7.$$

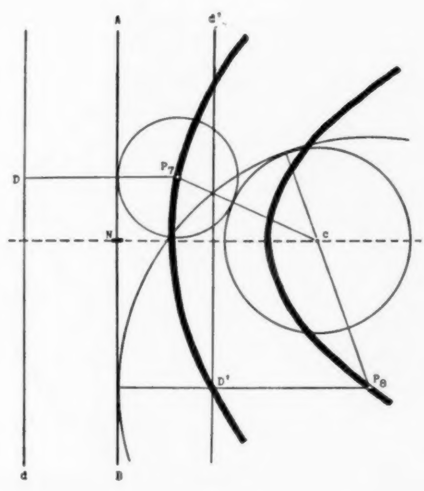


FIG. 5

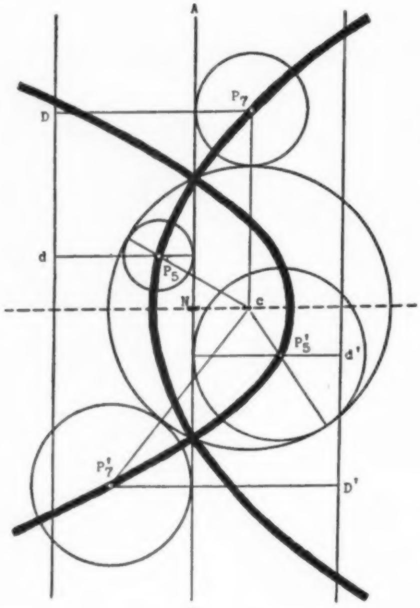


FIG. 6

Thus P_7 generates the parabola with focus at c, axis cN, and distance from focus to vertex equal to $1/2(cN+r)$. The directrix Dd and the focus c lie on opposite sides of AB.

Also from Figure 5:

$$cP_8 = \rho_8 - r = D'P_8.$$

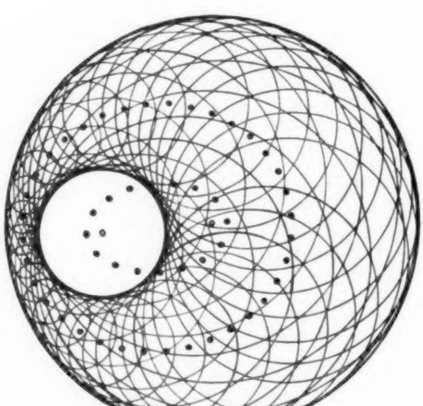


FIG. 7

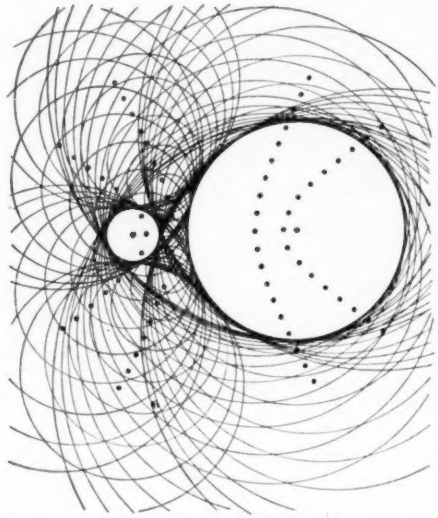


FIG. 8

Thus P_8 generates the parabola with focus at c, axis cN, and distance from focus to vertex equal to $1/2(cN-r)$. The directrix $D'd'$ and the focus c lie on the same side of AB.

In this case both parabolas open in the same direction.

(e) Case 5. Where $cN \leqq r$.

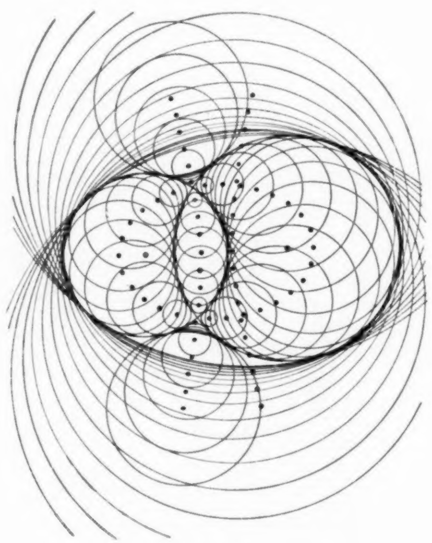


FIG. 9

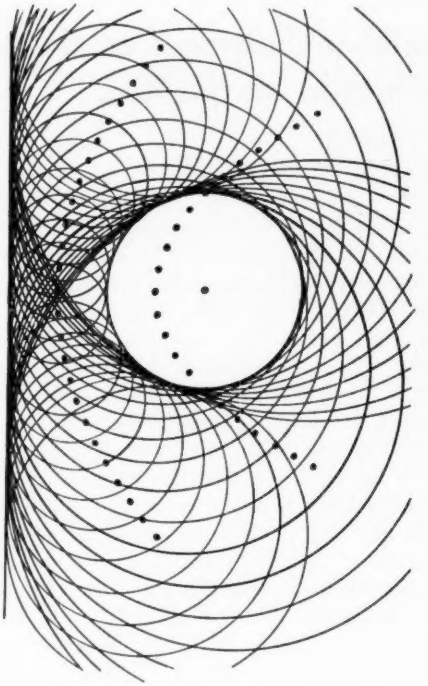


FIG. 10

In this case the two parabolas open in opposite directions.

SUMMARY

Two ellipses are generated when one of the given circles lies inside of the other (Fig. 7); two hyperbolas are generated when one of the given circles lies outside of the other (Fig. 8); an ellipse and a hyperbola are generated when the two given circles intersect (Fig. 9). The asymptotes of the hyperbolas are defined by the perpendicular bisectors of the line segments tangent to the two given circles.

As the radius of one of the given circles becomes infinite two parabolas are generated, opening in the same direction if the two given circles do not intersect (Fig. 10) and opening in opposite directions if they do intersect (Fig. 11).

Interesting limiting cases occur when the two given circles are tangent or concentric.

From Figure 6:

$$cP_5 = r - \rho_5 = dP_5 \quad \text{and} \quad cP_7 = r + \rho_7 = DP_7.$$

Thus P_5 and P_7 generate the parabola with focus at c, axis cN, and distance from focus to vertex equal to $1/2(r+cN)$. The directrix Dd and the focus c lie on opposite sides of AB.

Also from Figure 6:

$$cP_5' = r - \rho_5' = d'P_5' \quad \text{and}$$

$$cP_7' = r + \rho_7' = D'P_7'.$$

Thus P_5' and P_7' generate the parabola with focus at c, axis cN, and distance from focus to vertex equal to $1/2(r-cN)$.

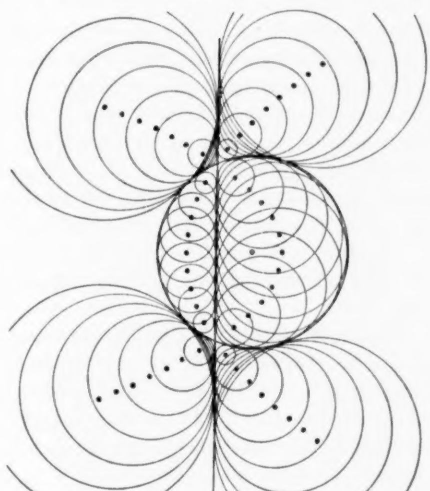


FIG. 11

A Graphical Method Useful in Solving Certain Algebraic and Trigonometric Inequalities

By Howard C. Bennett
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The method herewith presented has been used for the last two years in several courses in college algebra. The results have shown it to be an effective and simple method for solving a large number of conditional inequalities. The method presupposes only a few fundamentals of algebra. It is presumed that all students are already familiar with the rules for signs both in multiplication and division. For the purpose of this method it is only necessary to recall that an odd number of signs in either the multiplication of factors or in division yields a negative result, and that all positive signs, or an even number of negative signs, yield a positive result. For algebraic problems the only additional item needed is a familiarity with the usual method of representing all real numbers on the number scale. Following the customary procedure, on a line with an arbitrarily chosen and convenient unit of measurement, an arbitrary point is taken as the zero point. Positive numbers are numbered to the right and negative numbers to the left. If a is greater than b, it is on the right of b.

Below are given a few illustrative examples of the types usually given with their graphic solutions. It will be observed that when once the method is understood the solution is as simple for a large number of factors as it is for a few.

Example 1.

Find the values of x which satisfy $(x-1)\ (x+3)>0$.

As 1 is the critical value for the first factor, for all values of x to the right of that point this factor is positive and negative for all values of x to the left.

Since -3 is the critical value of the second factor, that factor is positive for all x to the right of the point and negative to the left as shown below in Figure 1. Dotted lines represent the interval in which the factor is negative, and solid lines the interval in which it is positive. According to the rule of signs the function will be positive; that is, will satisfy the inequality only where there are no dotted lines or an even number of such lines.

We note from the above that for all values of x to the right of 1 both factors are positive; therefore the product is positive and the inequality is satisfied. In the interval $-3<x<1$ there is one dotted line and, therefore, the inequality is not satisfied. For all values of x to the left of -3 there are 2 dotted lines and the inequality is satisfied. The results are tabulated below

$$x<-3, \quad x>-1.$$

The advantage of the second form is that it shows more clearly that there are two distinct intervals.

Example 2.

Find the values of x which satisfy

$$\frac{(x+2)(x-3)}{x+5}<0.$$

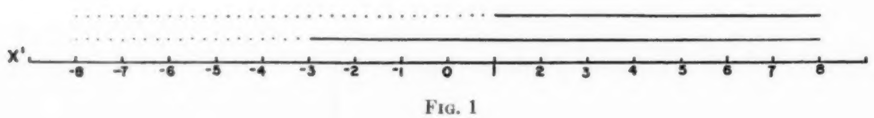

Fig. 1

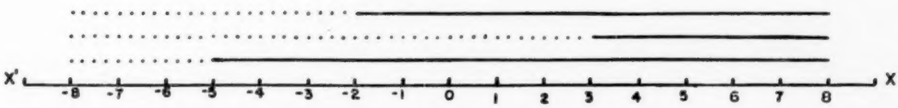

Fig. 2

The critical values -2, $+3$, and -5 with the corresponding intervals for each factor are represented in Figure 2.

In the interval $x<-5$ all three factors are negative and therefore the inequality is satisfied.

In the interval $-5<x<-2$ two are negative and it is not satisfied.

In the interval $-2<x<+3$ one factor is negative and two are positive; therefore the inequality is satisfied.

For $x>3$ all three factors are positive, therefore the inequality is not satisfied.

The tabulated results are

$$x<-5$$

$$-2<x<+3.$$

Example 3.

Find the values of x which satisfy

$$\frac{(x-2)^2}{(x-4)(x+3)}<0.$$

In the interval $x<-3$ all four factors are negative, and the inequality is not satisfied (Fig. 3). In the interval $-3<x<2$, three factors are negative and one is positive, hence the inequality is satisfied. In the interval $2<x<4$, one factor is negative so the inequality is still satisfied. In the interval $x>4$, all four factors are positive, thus the inequality is not satisfied. This problem includes the case of a double root. When care is taken to represent a double root by two different lines, the chart indicates immediately that the function still satisfies the inequality on the other side of the critical value.

The results shown are

$$-3<x<2$$

$$2<x<4.$$

This method is especially helpful in the graphing of a function that can be expressed in the form of factors or in the form of a quotient of factors. It is assumed that the student is familiar with Cartesian coordinates and knows that when a function is positive its graph is above the x-axis and when it is negative the graph is below the x-axis. If the student also knows that when the function equals zero the graph touches the x-axis and that when the denominator of a fraction becomes zero the graph is discontinuous, the general outline of the graph of a function can be easily sketched. Below is given an illustrative example.

Example 4.

Given

$$f(x)=\frac{x(x+2)}{(x-4)(x+1)}\cdot$$

State

(a) In what interval or intervals the graph is above the x-axis.

(b) In what interval or intervals it is below the x-axis.

(c) At what points it crosses the x-axis or is tangent to it.

(d) For what values of x it is discontinuous.

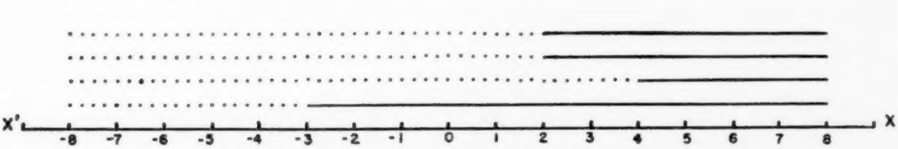

Fig. 3

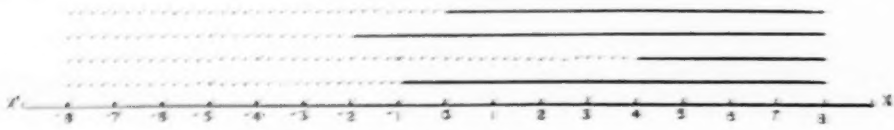

FIG. 4

(a) In the interval $x < -2$, all four are negative. In the interval $-1 < x < 0$, two are negative. In the interval $x > 4$, all four are positive.

(b) In the interval $-2 < x < -1$, there are three negative factors. In the interval $0 < x < 4$, there is one negative factor.

(c) The numerator becomes zero at $x = 0$ and $x = -2$. At each of these points the sign changes so the curve crosses the axis.

(d) The curve is discontinuous at $x = 4$. To the left of the point it is negative, to the right positive. The curve is also discontinuous at $x = -1$. To the left it is negative and to the right positive.

TRIGONOMETRIC INEQUALITIES

In order to use this method with trigonometric inequalities, it would be assumed, of course, that the student is already familiar with the variations in the numerical values of the trigonometric functions. In a college algebra course that does not require trigonometry as a prerequisite, it would not be possible to use this method. However, in courses that do presuppose a knowledge of trigonometry, we have found it to be very useful. An illustrative example is given below.

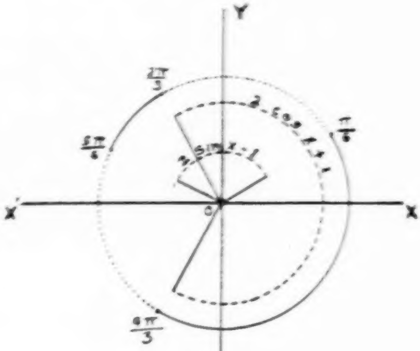

FIG. 5

EXAMPLE 5.

Find the values of x which satisfy

$$(2 \sin x - 1)(2 \cos x + 1) > 0$$

for values of x in the interval $0 < x < 2\pi$.

The critical value of the first factor occurs when $\sin x = \frac{1}{2}$. The related angle is $\pi/6$. This factor is positive when $(\pi/6) < x < (5\pi/6)$. As the critical value of the second factor occurs when $\cos x = -\frac{1}{2}$, the related angle is $\pi/3$. This factor is positive when $0 < x < 2\pi/3$ and $4\pi/3 < x < 2\pi$, the inequality is satisfied when $(\pi/6) < x < (2\pi/3)$ and $(5\pi/6) < x < (4\pi/3)$ as shown by the dotted arc on the outermost circle. Solid line arcs show where it is not satisfied.

In order to restrict the above explanation to the essentials of the method, illustrations were limited to problems already expressed in the form of products of factors. This limitation is not essential. A wide variety of problems may be included in which the student will be required to perform the necessary algebraic operations to express a function in the desired form. In the case of a quadratic expression, it is not necessary that the roots be rational. A few illustrations will suffice to show some of the various forms of problems which may be included.

EXAMPLE 6.

Find the values of x which satisfy the inequality

$$\frac{(x^3 - 4x^2)}{(x^2 - 5x + 6)} > 0.$$

This can readily be changed to

$$\frac{x^2(x-4)}{(x-3)(x-2)} > 0.$$

Noting that x is a double root, the prob-

lem is easily set up in the graphic form for a solution.

EXAMPLE 7.

Find the values of x satisfying

$$\frac{x^2-5x-3}{x^2-2x-5}<0.$$

The critical values of x in the numerator found by using the quadratic formula are $(5\pm\sqrt{37})/2$. The critical values of x in the denominator are $1\pm\sqrt{6}$. These irrational numbers can be closely approximated on the number scale. Then the problem can be solved as shown above.

EXAMPLE 8.

Find the values of x which make $f(x)$ positive, if $f(x)=x^2-5x+3$.

The critical values of x for this function by the quadratic formula are found to be $(5\pm\sqrt{13})/2$. By marking points representing these two numbers on the scale, the solution is easily found.

General Electric Fellowships for Secondary School Teachers of Mathematics for Summer of 1953

During the summer of 1953, 100 high school teachers of mathematics from twenty-four states will take advanced college training on General Electric Fellowships at Rensselaer Polytechnic Institute, Troy, New York and at Purdue University, Lafayette, Indiana.

Each College selects its candidates based on their qualifications and credentials and conducts the six-week college course which can be taken for credit toward an advanced degree. General Electric participation, beyond underwriting all expenses associated with each six-week program (tuition, board, lodging, fees and travel expenses) consists essentially of placing certain facilities and personnel at the disposal of the Colleges. Some General Electric engineers and scientists are requested to address the Fellows on their special fields. Also, the Company arranges trips to its laboratories and plants to show the teachers how mathematics or science is used in various operating areas of a typical American business and to observe scientists and engineers at work as well as to see manufacturing processes and a broad variety of modern machines and equipment. The Company feels that this represents a good investment as it should help high school teachers to do a better job of teaching boys and girls in the fields of science and mathematics. The six-week college course brings them up-to-date in their particular field, while the trips provide the opportunity to observe practical uses for the subject matter they are teaching. Living with other teachers from many states during a six-week period also stimulates enthusiasm for their work.

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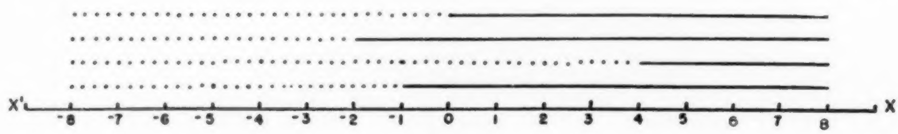
This will be the first year that the program will be given at Purdue University and will begin on June 22 and continue through August 1. The three courses to be included in this program here will be: Applied Mathematical Analysis, Experimental Techniques in Mathematics, and The Development of Mathematics. The objectives of the program are: (1) To stimulate enthusiasm for courses in mathematics on the part of teachers and, through them, on the part of students and officers of public instruction;.(2) To familiarize teachers with some of the important applications of mathematics; (3) To furnish teachers with mathematical information which may be of assistance in their teaching and in the guidance of their students; (4) To foster growth in the mathematical knowledge and background of the participants, and (5) in general, to aid these teachers in every possible way to make their secondary school mathematics courses of vital interest to the students. Applicants must be teaching in secondary schools in one of the following states: Illinois, Indiana, Iowa, Kentucky, Michigan, Minnesota, Missouri, Ohio, Tennessee, West Virginia, Wisconsin. Special forms for application may be obtained by writing to Professor Ralph Hull, Head, Department of Mathematics, Purdue University, Lafayette, Indiana and must be returned not later than March 2, 1953.



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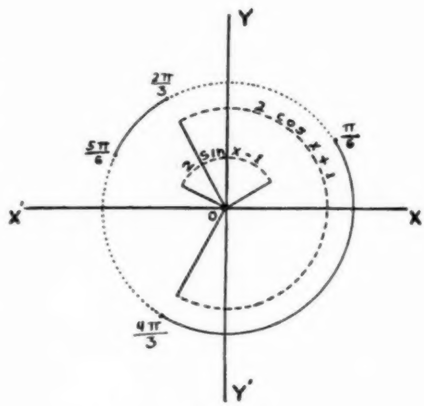

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DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are invited to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Devices for a Mathematics Laboratory to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

TRIGONOMETRIC FUNCTIONS DEVICE

All of us talk much about the usual trigonometric functions being "circular" functions. This device shows the relationship clearly. As the circle is turned indicating tabs continuously show values for the sine, cosine, and tangent (Fig. 1). The reader will find that this device is a very helpful one in introducing the functions of the general angle and in finding the values of the functions when graphing.

Following is a list of the materials needed to produce the device:

1 piece of plywood	$\frac{1}{4}''\times 2'\times 2'$
2 pieces of plywood	$\frac{1}{4}''\times 1'\times 1'$
2 wooden dowels	$1''\times 1\frac{1}{2}''$ long
1 piece of pine	$1''\times 1''\times 8''$
1 piece of pine	$2''\times 1''\times 8''$
1 piece of thick paper	$1'\times 1'$
Wood glue and wood screws.	

The procedure described in the remainder of this article will be found helpful in carrying out the details of construction.

Part (1) should be made first (Fig. 2). It may be made of either masonite or $\frac{1}{4}''$ plywood. It consists of a large square 2′ on a side with a circular hole 9″ in diameter cut from the center and three slots cut out according to the dimensions shown in the diagram of part (1). The 9″ circle cut from the center should be removed intact because it is needed in assembling the device.

Next cut out the two 10″ circles marked (2). On one side of each of these circles fasten a piece of 1″ dowel $1\frac{1}{2}''$ long. One of these dowels is the handle on the "Front View," and the other is part (5) shown on the "Rear View" (Fig. 3). On the second side of one of these 10″ circles fasten the 9″ circle which was cut from part (1) above by using wood screws; the centers must coincide. Next cut out and glue a piece of thick paper over the exposed side of the 9″ circle. Then place this unit of two circles on part (1) so that the 9″ circle is inside the hole from which it was originally cut, and fasten the second 10″ circle to the 9″ circle from the rear side of part (1). The three circles thus assembled can be turned together as a unit and should clear part (1) nicely because of the sheet of thick paper between the two 10″ circles.

Next cut out the two parts marked (3); be especially careful not to make the slots wider than 1″. Parts marked (7) should be cut from a piece of tin; three identical pieces are required. These are the indicator tabs. The positions of these tabs on the number (3) parts may be determined by arranging the number (3) parts as shown on the "Rear View" (Fig. 3). By turning the disc so that the rear dowel is first on the right and then on the left the point at which the cosine tab indicator must be fastened may be determined in such a way that it does not touch the ends of the slot in which it moves. By moving the rear dowel to the extreme top and bottom the required positions of the sine and tangent tabs can be located. Fasten the tab indicators to the parts (3) at the points located with small screws or brads. Note from the "Rear View" diagram that the

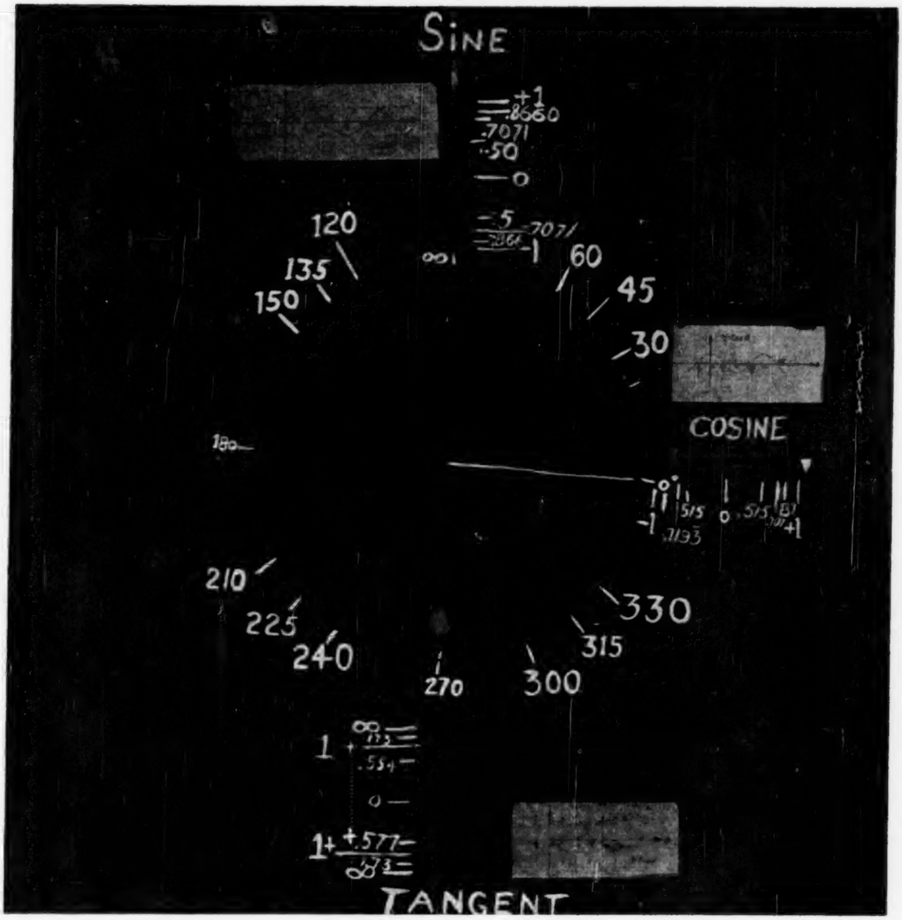


FIG. 1

vertical part (3) is placed on the bottom of the horizontal part (3).

Next cut out parts (4) and (6); be sure to make the grooves wide enough to allow parts (3) to slide through them freely. Fasten parts (4) and (6) to the back side of part (1) as shown on the "Rear View" diagram. A word of caution must be added here. Before screwing parts (4) and (6) down tight, check to see that they are in close enough so that parts (3) will not run out of the grooves when the disc is turned.

Now locate the angular values on the front side of part (1) and paint them on (Fig. 1). To locate the reference line on the disc turn it until the cosine tab is at the extreme right (+1) and then paint a straight line from the center of the disc out to the 0° mark. Locate the functional values of the cosine along the slot on the right by recording the positions of the tab indicator as the disc is turned through 360°. Paint these on the face of part (1) as before. The functional values of the sine and tangent may be located in a similar manner.

By painting the entire device in bright colors more attention may be had from the student. If the curves of the functions are taught it is a good idea to paint them near their respective functions. (In Figure 1

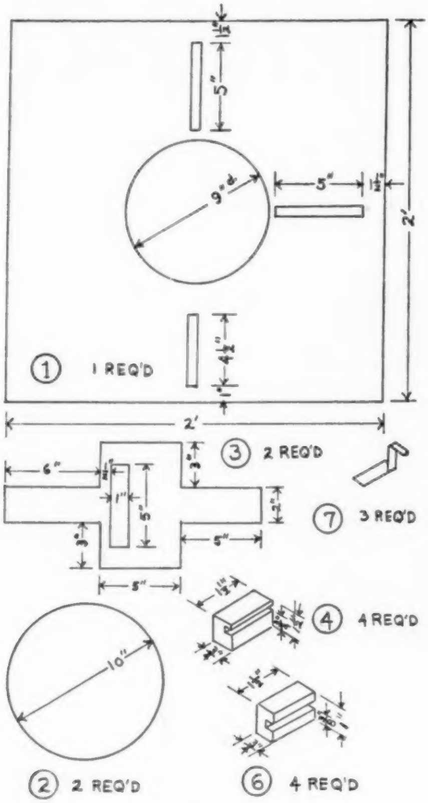


Fig. 2

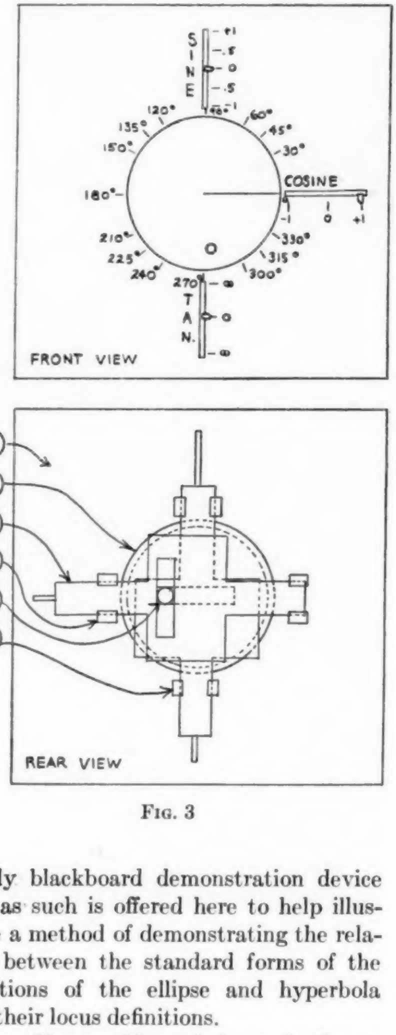


Fig. 3

the different curves have been drawn on small sheets of cross section paper and pasted near their respective functions.) One more point seems worth mentioning. The reader will see that when the disc is turned the indicator tab for any function moves back and forth across its zero functional value in the same way that the curve does.

Luther Stone
State Teachers College
Millersville, Pennsylvania

Blackboard Locus Drawing Device

This note is included in this section with some reluctance because it may duplicate, at least in part, ideas that have appeared in this and other journals before. However, the device is a rather handy blackboard demonstration device and as such is offered here to help illustrate a method of demonstrating the relation between the standard forms of the equations of the ellipse and hyperbola and their locus definitions.

Any ellipse and hyperbola can be drawn on the blackboard in accordance with their locus definitions by using two rubber suction cups as foci and a piece of chalk tied to a piece of string. The only problem encountered in this connection is that of improvising a scheme for holding the string. A junior student in the department editor's class met this problem by placing small round metal sewing machine bobbins over the screws in the tops of the suction cups and fastening them down by pouring solder into the cores around the

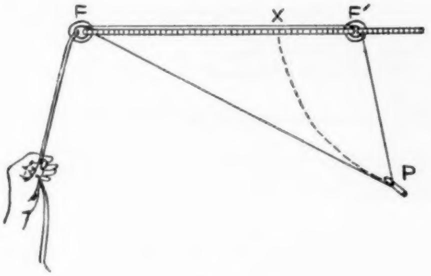


FIG. 4

screws (Fig. 4). (Suction cups with screws molded into the rubber may be obtained commercially.) As an added feature an iron rod may be soldered to the top of the bobbin at F and slipped through a screw-eye soldered into the top of the bobbin at F'. If the rod is divided into centimeters a tidy arrangement results for measuring the distance between the foci.

Suppose the teacher desires to demonstrate that the ellipse described by the equation,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$$

can be produced by a point which moves so that the sum of its distances to two fixed points, F and F', is constant. How far apart must the foci be located and what must be the length of the loop of string that is to be placed around the two foci? If the distance between the foci is designated as $2c$, where $a^2-b^2=c^2$, then the loop of string must be $2c+2a$ units in length. The reader is undoubtedly familiar with these facts and determination of the actual measurements needed for drawing some particular ellipse is only a matter of substituting the appropriate values given for a and b.

The manner of drawing the hyperbola with string and fixed foci may not be so well known. According to the usual definition the hyperbola may be defined as the locus of a point which moves so that the difference of its distances to two fixed points is constant. Consider the hyperbola described by the following equation—

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$$

In order to draw this curve tie a piece of chalk somewhere near the middle of a long piece of string. Grasp the two loose ends in the left hand and the piece of chalk in the right hand. Place the closed loop around the bobbin at F' and then lift both strands held by the left hand across the top of the bobbin at F (Fig. 4). For the equation given above the transverse axis is $2a$ units in length. Accordingly the string must be adjusted so that the chalk will be at X in such a position that $FX-F'X=2a$. Then the lower half of the right hand branch can be drawn simply by pulling the chalk downward. With this suggestion as a beginning the reader should have little difficulty in producing the remainder of the curve. If the distance between the foci is again designated as $2c$, then the length of this segment in centimeters for any particular hyperbola may be determined from the relation $c^2=a^2+b^2$, a fact with which the reader is assumed familiar.

For high school students the intuitive approaches outlined above generally suffice, and they help illustrate how the equations and locus definitions are related without going into the developments presented in analytic geometry.

THE MATHEMATICS LABORATORY
Monroe High School

HAVE YOU SEEN?

In *The Journal of Engineering Education*, October 1952

"A Procedure for Teaching Fundamentals" by Frederick W. Ross

"Is the Traditional Program in College Mathematics Adequate for the Training of Prospective Engineers and Scientists?" by Kenneth L. Noble

"Right Triangles with Integral Sides" by F. C. Bragg

MATHEMATICAL RECREATIONS

Edited by Aaron Bakst
135-12 77th Avenue, Flushing 67, N. Y.[1]

In the January 1952 issue of *The Mathematics Teacher* (Volume XLV, pages 72 and 73) the following problem was discussed: *The extreme left digit of a number is 7. When this digit is transposed to the extreme right, the original number is divisible by 3.* Some very interesting results were obtained:

1. The solution of this problem does not require any special masteries and skills.
2. Such a problem is within the scope of difficulty of the pupils who are in command of arithmetic division of two-place numbers by a one-place number.
3. The number in question may be considered as a repeating decimal.
4. The first period of this number is the *minimal* number.

The problem proposed in the January issue of The Mathematics Teacher permits generalization. In the most general form, this problem may be stated as follows: "A number has a digit a on its extreme left. This digit a is transposed to the extreme right. When this transposition has been performed, the new number is equal to $1/n$ of the original number. Obtain the general expression of this number." The problem in this formulation is certainly beyond the scope of elementary mathematics. However, let it be hoped that some of the students of pure mathematics will try to sharpen their teeth and claws on it.

[1] The material presented here appears in the forthcoming book by the editor of this department, *Mathematical Games and Recreations* (D. Van Nostrand Co., Inc., New York). Copyright, 1953, by D. Van Nostrand Co., Inc.

Nevertheless, specific solutions of this problem do not require anything else than that which was indicated above. We shall state a set of solutions for the following values of a and n.

$$a = 1, 2, 3, 4, 5, 6, 7, 8, \text{ and } 9$$

and

$$n = 1, 2, 3, 4, 5, 6, 7, 8, \text{ and } 9.$$

The method of the solution of the specific problem-situations was indicated in this department in the January 1952 issue of The Mathematics Teacher. We will indicate the respective solutions for the values of n stated above.

For $n=1$. Any number with the same sequence of digits, as, for example, 2,222,222, 333,333, 555,555, 77,777, and so on.

For $n=2$.

105,263,157,894,736,842,
210,526,315,789,473,684,
315,789,473,684,210,526,
421,052,631,578,947,368,
526,315,789,473,684,210,
631,578,947,736,842,105,
736,842,105,263,157,894,
842,105,263,157,894,736,
947,368,421,052,631,578.

For $n=3$.

1,034,482,758,620,689,655,172,413,793,
2,068,965,517,241,379,310,344,827,586,
3,103,448,275,862,068,965,517,241,379,
4,137,931,034,482,758,620,689,655,172,
5,172,413,793,103,448,275,862,068,965,
6,206,896,551,724,137,931,034,482,758,
7,241,379,310,344,827,586,206,896,551,
8,275,862,068,965,517,241,379,310,344,
9,310,344,827,586,306,896,551,724,137.

For $n=4$.

102,564,
205,128,
307,692,
410,256,
512,820,
615,384,
717,948,
820,512,
923,076.

For $n=5$.

102,040,816,326,530,612,244,897,959,183,673,469,387,755,
204,081,632,653,061,224,489,795,918,367,346,938,877,551,
306,122,448,979,591,836,734,693,877,551,020,408,163,265,
408,163,265,306,122,448,979,591,836,734,693,877,551,020,
510,204,081,632,653,061,224,897,959,183,673,469,387,755,
612,244,897,959,183,673,469,387,755,102,040,816,326,530,
714,285,
816,326,530,612,244,897,959,183,673,469,387,755,102,040,
918,367,346,938,775,510,204,081,632,653,061,224,489,795.

For $n=6$.

1,016,949,152,542,372,881,355,932,203,389,830,508,474,576,271,186,440,677,966,
2,033,898,305,084,745,762,711,864,406,779,661,016,949,152,542,372,881,355,932,
3,050,847,457,627,118,644,067,796,610,169,491,525,423,728,813,559,322,033,898,
4,067,796,610,169,491,525,423,728,813,559,322,033,898,305,084,745,762,711,864,
5,084,745,762,711,864,406,779,661,016,949,152,542,372,881,355,932,203,389,830,
6,101,694,915,254,237,288,135,593,220,338,983,050,847,457,627,118,644,167,796,
7,118,644,167,796,610,169,491,525,423,728,813,559,322,033,898,305,084,745,762,
8,135,593,220,338,983,050,847,457,627,118,644,067,796,610,169,491,525,423,728,
9,152,542,372,881,355,932,203,389,830,508,474,576,271,186,440,677,966,101,694.

For $n=7$.

1,014,492,753,623,188,405,797,
2,028,985,507,246,376,811,594,
3,043,478,260,869,565,217,391,
4,057,971,014,492,753,623,188,
5,072,463,768,115,942,028,985,
6,086,956,521,739,130,434,782,
7,101,449,275,362,318,840,579,
8,115,942,028,985,507,246,376,
9,130,434,782,608,695,652,173.

For $n=8$.

1,012,658,227,848,
2,025,316,455,696,
3,037,974,683,544,
4,050,632,911,392,
5,063,291,139,240,
6,075,949,367,088,
7,088,607,594,936,
8,101,265,822,784,
9,113,924,050,632.

For $n=9$.

10,112,359,550,561,797,752,808,988,764,044,943,820,224,719,
20,224,719,101,123,595,505,617,977,528,089,887,640,449,438,
30,337,078,651,685,393,258,426,966,292,134,831,460,674,157,
40,449,438,202,247,191,011,235,955,056,179,775,280,898,876,
50,561,797,752,808,988,764,044,943,820,224,719,101,123,595,
60,674,157,303,370,786,516,853,932,584,269,662,921,348,314,
70,786,516,853,932,584,269,662,921,348,314,606,741,573,033,
80,898,876,404,494,382,022,471,910,112,359,550,561,797,752,
91,111,235,955,056,179,775,280,898,876,404,494,382,022,471.

Numbers which possess the properties described above may be obtained for systems of numeration other than the decimal.

Thus, for example, in the binary system of numeration (with the base 2) the smallest number which possesses the property discussed above is 11.

In the system of numeration with the base 3 we have the following numbers:

For $n=1$, 11, 22,

For $n=2$, 1,012 and 2,101.

In the next issue of THE MATHEMATICS TEACHER such numbers in systems of numeration other than the decimal will be discussed in detail.

1953 Metropolitan New York—MAA Mathematics Contest

The Fourth Annual Contest sponsored by the Metropolitan New York Section of the Mathematical Association of America will be held on Thursday morning, May 14, 1953. Any high school in the United States or in Canada is privileged to enter.

8692 students took the 1952 test and represented 295 high schools distributed as follows:

Area I. The Metropolitan New York Section .... 199 schools

Area II. The State of Connecticut (Contest was scheduled during spring vacation in Connecticut) .... 27 schools

Area III. The State of Oregon (Sponsored by the Mathematics Department of the University of Oregon) .... 35 schools

Area IV. Any other state in which the MAA is not conducting a contest. The entries were from Alabama, Arkansas, California, Colorado, Delaware, Florida, Georgia, Iowa, Illinois, Indiana, Kentucky, Louisiana, Maine, Maryland, Massachusetts, Michigan, Montana, Nebraska, New Hampshire, North Dakota, Ohio, Pennsylvania, South Carolina, South Dakota, Tennessee, Texas, Vermont, Washington and West Virginia .... 39 schools

Area V. The Province of British Columbia (Sponsored by the Mathematics Department of the University of British Columbia) .... 6 schools

Certificates of Merit were awarded to the highest ranking student in a county provided his score was over 80 points. Awards were as follows:

Rank	*Name*	*School*	*County*	*State*	*Score*
1	Paul Henry Monsky	Brooklyn Tech. H. S.	Kings	N. Y.	150
2	Richard Dolen	Bronx H. S. of Science	Bronx	N. Y.	135
3	Barbara Michelle White	Marymount School	N. Y.	N. Y.	124
4	Max Abraham Plager	Hackensack H. S.	Bergen	N. J.	114
	Geraldine Anne Ferraro	Marymount Sec. School	Westchr.	N. Y.	114
5	James Bernard Frazer	The Choate School	N. Haven	Conn.	92
6	Robert Andrew Schade	E. Rockaway H. S.	Nassau	N. Y.	91
	John Hugh Maltby	Mahopac High School	Putnam	N. Y.	91
7	Lawrence Gordon Dodd	Franklin High School	King	Wash.	85
8	Ellen Marie Dolganos	Stevens Hoboken Acad.	Hudson	N. J.	84
	Robert Donald Marcus	Forest Hills H. S.	Queens	N. Y.	84
9	Stephen T. Schlager	Stamford High School	Fairfield	Conn.	82
10	Norman Alison Riley	Linden High School	Union	N. J.	81

The five schools which had total scores of over 300 points for their three highest contestants were: Brooklyn Technical H. S., Bronx H. S. of Science, Marymount School of New York City, Abraham Lincoln H. S. of Brooklyn and the Marymount Secondary School of Tarrytown, N. Y. Next ranking schools included the Midwood, James Madison, and Erasmus Hall High Schools of Brooklyn, East Rockaway (N. Y.) H. S., The Choate School of Wallingford (Conn.), Forest Hills (N. Y.) H. S., Mamaroneck (N. Y.) H. S., Franklin H. S. of Seattle (Wash.), Bronxville (N. Y.) H. S., Stevens Hoboken (N. J.) Academy, North Arlington (N. J.) H. S., Lynbrook (N. Y.) H. S., Teaneck (N. J.) H. S., Hackensack (N. J.) H. S., and New Utrecht H. S. of Brooklyn. Additional ranking schools not located in the Metropolitan New York Area were: Stamford (Conn.) H. S., Tigard (Oregon) H. S., Loomis School of Windsor (Conn.), Lincoln (Nebr.) H. S., Staples H. S. of Westport (Conn.), Deering H. S. of Portland (Maine), Windham H. S. of Willimantic (Conn.), Salem (Oregon) H. S., Central H. S. of Aberdeen (S. D.) and the Austin (Texas) H. S.

Some sample copies of the questions used in the 1951 and 1952 contests are still available and comments and criticisms of the 1952 questions are invited by the Committee. Requests for information should be addressed to Professor W. H. Fagerstrom, Chairman of the Committee on Prizes and Awards, Metropolitan New York Section of the Mathematical Association of America, The City College of New York, Convent Avenue and 139th Street, New York 31, New York.

The President's Page

CO-OPERATION WITH OTHER PROFESSIONAL GROUPS

ONE OF THE WAYS in which the National Council can contribute to the improvement of mathematics teaching is through the promotion of and participation in co-operative and constructive planning among teachers of mathematics and of other subjects and school administrators within a school system and on a county, state, regional, and national scale. The fact that National Council programs and many of our publications, as well as the programs and publications of the Affiliated Groups, are planned for teachers at all levels of instruction is continuing to assist in improved understanding among elementary, secondary, and college teachers of mathematics. In the May, 1951 number of THE MATHEMATICS TEACHER a report was made on the co-operation and affiliation of the Affiliated Groups with education associations in the areas which they represent. The advantages which can come to the National Council through affiiliation with the National Education Association will be reported on this page at a later date.

Activities co-operatively sponsored by our Affiliated Groups and the Sections of the Mathematical Association of America were reviewed in the February, 1952 number of THE MATHEMATICS TEACHER. While the National Council cannot take credit for this important work, it should be understood by all that the National Council desires to encourage and promote these very desirable associations. Regional co-operation probably can have a more direct and beneficial effect than similar efforts on a national scale. However, National Council activites jointly sponsored with the Mathematical Association can and should become an even more important part of the Council's total program directed toward the improvement of mathematics instruction at all levels.

Since its organization in 1920 the National Council has sought opportunities to co-operate with the Mathematical Association. The best known and most fruitful instance of this co-operation is probably the work of the Joint Commission of the two organizations which sponsored the publication of our Fifteenth Yearbook, "The Place of Mathematics in Secondary Education." Too infrequently the groups have held meetings in the same place so that members of each could take advantage of the programs of the other. Among the most productive of recent committees sponsored jointly by the two organizations have been the Advisory Council on Mathematics in Industry, Science, Business and Engineering, of which W. W. Rankin of Duke University has been the chairman, and the Joint Committee on a Symposium on Teacher Education in Mathematics which was responsible for planning the Symposium held at The University of Wisconsin in August, 1952. The Advisory Council is active at the present time and plans are underway for a new committee to plan some kind of jointly sponsored study as a follow-up of the Symposium.

Co-operation with other professional groups has not been limited to education associations and the Mathematical Association. As illustration, the National Council has a representative on three important professional committees. These Committees and our representatives are:

A.A.A.S. Co-operative Committee on Teaching Science and Mathematics—George Hawkins, Lyons Township High School and Junior College, LaGrange, Illinois.

The membership of the Committee includes representatives of sixteen professional organizations in science and mathematics. The Committee is sponsored by the American Association for the Advancement of Science. Dr. Morris Meister, principal of the Bronx High School of Science, New York City, is chairman of the Committee.

Mathematics Policy Committee—E. H. C.

(*Continued on page 98*)

Nominations for 1953 N.C.T.M. Ballot

The 1953 Nominating Committee presents the persons listed below as candidates for the designated offices on the Board of Directors. Two candidates are presented for each office. The term of office for Director is three years, and for Vice-President is two years.

The Nominating Committee wishes to thank the members of the Council who made suggestions to the Committee. Since not all of the names submitted could be used on the present ballot, it is suggested that names omitted may properly be suggested to the Nominating Committee for 1954.

EDITH WOOLSEY, *Chairman*
HARRY W. CHARLESWORTH
AGNES HERBERT
LENORE JOHN
F. LYNWOOD WREN

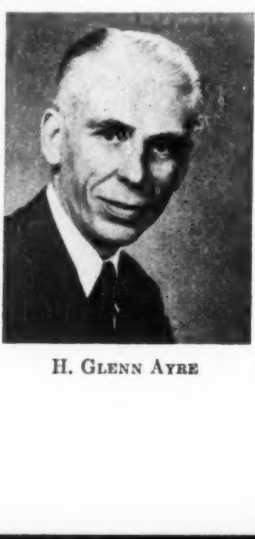
Vice President—College Level

AYRE, H. GLENN, Professor of Mathematics, Head of the Department of Mathematics and Director of the General College Division, Western Illinois State College, Macomb, Illinois.

Ed. B., Southern Illinois University; S.M., University of Michigan; Ph.D., Peabody College for Teachers.

Activities in NCTM include: Member of Algebra Com. for 17th Yearbook; Secy. of Symposium on Teacher Educ. in Math., Univ. of Wis., 1952; Member, Publicity Com., 1952–.

Instructor in rural school, Marion County, Ill.; H.S., Carterville, Ill.; H.S., Benton, Ill.; H.S., Waukegan, Ill.; Supervision of Student Teaching in Mathematics and Instructor in Mathematics, Western Illinois State College, Macomb, Ill.; Professor of Mathematics, Head of the Department of Mathematics and Director of the General College Division, Western Illinois State College, Macomb, Ill. Member of CASMT (Chmn., Math. Sec., 1941; Chmn., Jr. Col. Sec., 1947); MAA (V.-Chmn. 1951–52, Chmn., 1952–53, Ill. Sec.; Joint Com. with NCTM to Study Teacher Training in Math.); NEA; Ill. Ed. Assn.; Ill. Council of Tchrs. of Math. (Exec. Council, 1948–52; Chmn., Sec. Conferences, 1951–53); Phi Delta Kappa; speaker and discussion leader for numerous institutes and workshops on mathematics education. Listed in *American Men of Science*, *Who's Who in Education*, and *Who's Who in Chicago and Illinois*. Author of *Basic Mathematical Analysis*, "An Analysis of Individual Differences in Plane Geometry," "An Analysis of the Performance of College Freshmen on Arithmetic," in *Western Illinois State College Quarterly*, and articles in THE MATHEMATICS TEACHER.



H. GLENN AYRE

HOUSTON T. KARNES

KARNES, HOUSTON T., Associate Professor of Mathematics, Louisiana State University, Baton Rouge, Louisiana.

A.B., A.M., Vanderbilt University; Ph.D., Peabody College for Teachers; summer sessions, University of Wisconsin and the University of Michigan.

Activities in NCTM include: La.-Miss. State Representative; Member of Planning Com. of 1952 Symposium on Tchr. Educ. in Math.; Co-editor of the Dept. "What is Going on in Your School?" of THE MATHEMATICS TEACHER; served on several committees of NCTM and MAA.

Professor Mathematics and Biology, Northwestern Junior College, Orange City, Ia., 1929–35; Professor of Mathematics and Dean of Men, Harding College, Searcy, Ark., 1935–36; Teacher of Mathematics and Department Head,

high schools, Nashville, Tenn., 1936–38; Associate Professor of Mathematics, Louisiana State University, 1938–. Member of MAA; Am. Math. Soc.; NEA; La. Educ. Assn.; AAUP; Pi Mu Epsilon (Associate Editor of *Pi Mu Epsilon Journal*); Kappa Mu Epsilon; Phi Delta Kappa; Kappa Delta Pi; Omicron Delta Kappa; Board of Trustees, Harding College; La.-Miss. Branch of NCTM (Recorder, delegate Assembly); Director of the La. State Univ. Math. Inst.; Chmn., Standing Committee of Mathematicians of La. for the Furtherance of Math. Educ. Listed in *American Men of Science*, *Who's Who in American Education* and the *Supplement of Who Knows and What*. Author of "Preparation of Teachers of Secondary Mathematics" and "Junior College Mathematics Curriculum Problems in View of the President's Report," THE MATHEMATICS TEACHER; and "Legislation vs. Education," *American Mathematical Monthly*.

MARY C. ROGERS

Vice-President—Junior High School Level

ROGERS, MARY C., Roosevelt Junior High School, Westfield, New Jersey.

B.S., State Teachers College, Mansfield, Pa.; graduate work, State Teachers College, Mansfield, Pa., Cornell University, New York University.

NCTM activities include: Member, Board of Directors, 1950–53; Chairman, Committee on Affiliated Groups, 1952–; Chairman, Speakers' Bureau, 1950–52; Chairman, Membership Honor Schools, 1948–53; New Jersey State Representative, 1942–; New Jersey Delegate, 1950 and 1951 Delegate Assemblies; Coordinator, Local Committees for Annual Meeting, 1940–41; Chairman, Hospitality for Annual Meeting, 1946–47; General Chairman of Local Committees for Annual Meeting 1952–53; Coordinator, Eastern Section Meeting, New York City, December 1949; Member, Committee on Contests and Scholarships, 1949–; Member, Publicity and Program Committees (Baltimore, Chicago, Madison, Gainesville, Pittsburgh meetings).

Vice-Principal and Teacher, Grades 6, 7 and 8, Eagles Mere, Pa.; Vice-Principal and Teacher of Mathematics, H.S., Meshoppen, Pa.; Teacher, H.S., Harford, Pa.; H.S., Westfield, Pa.; H.S., Milford, Pa.; Mathematics Instructor, State Teachers College, Mansfield, Pa.; Head of Mathematics Department, Roosevelt Jr. H.S., Westfield, N. J. Study Group Leader in Jr. H.S. Math., Duke Math. Inst., 1947; N.E. Inst. Tchrs. Math., 1950; Math. Conf., Univ. of Wis., 1952. N. J. St. Syllabus Com., 1933–35; N. J. St. Curriculum Com., 1948–50. Member, Assoc. Math. Tchrs., N. J. (Chmn., Com. on Revision of Const., 1939–40; Pres., 1940–41; Editor, *N. J. Math. Tchr.*, 1941–42; Secy.-Treas., 1941–; Mem. chmn., 1942–50; chmn., Jr. H.S. Research, Post-War Policies Com., 1947–49; Chmn., N. J. Math. Inst., 1952–); Section 10, Met. Group for the Experimental Study of Math. Educ.; Natl. Safety Council; Pi Lambda Theta. Author of articles in *The New Jersey Mathematics Teacher;* THE MATHEMATICS TEACHER; *National Safety Magazine; Rho Journal—Pi Lambda Theta.*

MILTON W. BECKMANN

BECKMANN, MILTON W., Teacher of Mathematics in Junior High School Grades and Supervisor of Mathematics, Teachers College High School, University of Nebraska; also Assistant Professor, Teachers College, University of Nebraska.

B.A. and M.A., University of Nebraska; additional study, University of Chicago; Ph.D., University of Nebraska.

Activities in NCTM include: Pres., Nebr. Sec., NCTM, 1940, 1950–52; General Chairman, 13th Christmas meeting.

Teacher of Junior High Mathematics, Kearney, Nebr. for five years; high school mathematics teacher; administrator of schools for seven years; Supervisor of Mathematics, Teachers College High School, University of Nebraska, 1949–. Life member, Nebr. St. Educ. Assn. (Pres., Dist. No. 4, 1948–49); member, Nebr. Schoolmasters Club (Secy.-Treas.); "N" Club; NEA; Phi Delta Kappa; AAUP; Capitol City Educ. Assn., Lincoln (Pres., 1952–53). Publications include: "Safety Education Needs Met Mathematically," "How Mathematically Literate is the Typical Ninth Grader After Having Completed Either General Mathe-

matics or Algebra?" *School Science and Mathematics;* "Efficiency of Automobile Brakes," *Nebraska Education Journal;* "Teaching Safety Through Mathematics," *Safety Education;* author of *Shop Mathematics,* correspondence course of Ext. Div., Univ. of Nebr.; "Is General Mathematics or Algebra Providing Greater Opportunity to Attain the Recommended Mathematical Competencies?" to be published in the *Journal of Educational Research.*

Howard Franklin Fehr

Additional Members of the Board of Directors

Fehr, Howard Franklin, Professor of Mathematics and Head of the Department of Teaching of Mathematics, Teachers College, Columbia University, New York, New York.

B.A., A.M., Lehigh University; Ph.D., Columbia University.

Instructor in Mathematics, H.S., Bethlehem, Pa.; H.S. for Boys, Reading, Pa.; South Side H.S., Newark, N.J.; Instructor and Professor of Mathematics, State Teachers College, Montclair, N. J., 1934–48; Teachers College, Columbia University, 1948–. Also Instructor at Newark College of Engineering and Rutgers College. Member, CASMT; MAA; AAAS; Am. Ed. Research Assn.; and other educational societies. Past president of Assn. of Math. Tchrs. of N. J. and Assn. of N. J. St. Tchrs. Colleges (3 terms). Listed in *Who's Who in America, American Men of Science, Who's Who in American Education.* Publications include *A Study of the Number Concept of Secondary School Mathematics; Secondary Mathematics, A Functional Approach for Teachers;* co-author of *Senior Mathematics for High Schools;* and numerous articles in *Teachers College Record,* The Mathematics Teacher, *School Science and Mathematics,* and other educational journals.

Johnson, Donovan A., Head, Mathematics Department and Assistant Professor of Education, University of Minnesota High School, Minneapolis, Minnesota.

B.S., M.A., Ph.D., University of Minnesota.

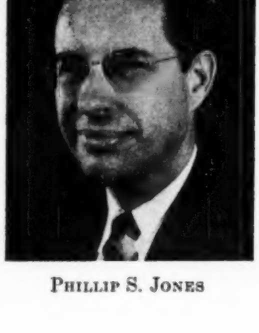
Activities in NCTM include: Member, Board of Directors, 1950–53; Co-editor of Dept., "Aids to Teaching" in The Mathematics Teacher; member, Multi-Sensory Aids Com.; Chmn., Prog. Com., 1951 summer meeting.

Science Teacher, Stillwater, Minn., 1933–36; Mathematics Teacher, Sheboygan, Wis., 1936–42; Instructor, Naval Training School, University of Minnesota, 1942–44; Head of Mathematics Department and Assistant Professor of Education, University of Minnesota High School, 1944–. Member of Math. Sec. of Minn.

Donovan A. Johnson

Educ. Assn. (Pres.); Minn. Council of Tchrs. of Math. (Member of Executive Board); CASMT (Member of Com. on Research in Math.); MAA (Chmn., Coordinating Com., Minn. Chap.); Phi Delta Kappa; NEA. Author: "An Experimental Study of the Relative Effectiveness of Certain Visual Aids in Teaching Geometry," *Journal of Experimental Education,* Mar., 1949; "Filmstrips in Mathematics," *Visual Review,* 1947; "Toward Living Mathematics," *See and Hear,* Mar., 1946; co-author: "Bibliography of Mathematics Films and Filmstrips," *School Science and Mathematics,* Nov. 1949; "The Future of Films in Mathematics," The Mathematics Teacher, April 1947; and "Vitalizing Geometry with Visual Aids," The Mathematics Teacher, Feb., 1940.

Jones, Phillip S., Assistant Professor of Mathematics and of the Teaching of Mathematics and Supervisor of Teaching Fellows in



Phillip S. Jones

Mathematics at the University of Michigan.

A.B., Teachers Life Certificate, Secondary School, A.M., Ph.D., University of Michigan.

Activities in NCTM include: Member of Committee on the Official Journal, Editor of the Dept. "Mathematical Miscellanea" and Associate Editor of THE MATHEMATICS TEACHER.

Teacher at Jackson, Mich. H.S., Jr. Col., and Adult Evening School; The Edison Institute of Technology; Univ. School, Ohio State University; U. S. Army ASTP, Navy V-12, ESMWTP, Reserve Officers Naval Architecture Program at University of Michigan. Member of: Mich. Council of Tchrs. Math.; CASMT; MAA (Secy.-Treas., Mich. Sec.); Am. Math. Soc.; Hist. of Sci. Soc.; AAAS; AAUP; Sigma Xi; and Phi Kappa Phi. Author of occasional articles in THE MATHEMATICS TEACHER, most recently, "Mathematical Preparation for College" in May 1952 issue (with P. D. Edwards and B. E. Meserve); three chapters in the Eighteenth Yearbook of the NCTM (Multi-Sensory Aids); also articles in *Scripta Mathematica; American Mathematical Monthly;* and *School Science and Mathematics.*

ELLA C. MARTH

MARTH, ELLA C., Chairman of the Division of Mathematics and Business Education, Wilson Teachers College, Washington, D. C.

A.B., Harris Teachers College; M.S., Ph.D., St. Louis University; Post-doctoral study at Washington University, St. Louis University.

Activities in NCTM include: Missouri State Representative, 1947–52; Local Chairman for the NCTM meeting with the NEA, July 1950, at St. Louis.

Elementary and Secondary Teacher of Science and Mathematics, St. Louis Public Schools; Instructor in Mathematics, Southeast Missouri State College; Assistant and Associate Professor, Dean of Women, Harris Teachers College. Consultant in Arithmetic, St. Louis Public Schools, 1945–51; Lecturer in Education, St. Louis University, summers 1945–47, spring 1947; Chairman of the Division of Mathematics and Business Education, Wilson Teachers College, Washington, D. C. Member: Am. Math. Soc.; MAA, NEA; Assn. for Supervision and Curriculum Development; ACEI (local officer for the St. Louis Chapter); AAUP (local secretary); Natl. Voc. Guidance Assn.; Sigma Xi; and Pi Mu Epsilon. Served as leader at Curriculum Conferences at St. Louis Univ.; study group leader, Duke Univ. Inst. Listed in *American Men of Science; Who's Who in American Education.* Publications include "Symmetry in Three Dimensions" in the *Mathematics Student* and "Further Properties of Garvin's F-Series" in the *Duke Mathematical Journal.*

PHILIP PEAK

PEAK, PHILIP, Assistant to the Dean, School of Education, and Head of the Mathematics Department, University School, Indiana University, Bloomington, Indiana.

B.A., Iowa State Teachers College, Cedar Falls, Ia.; M.S. University of Iowa.

NCTM activities include: Indiana State Representative; Member of Twenty-second Yearbook Committee, and Member of Research in Algebra Committee.

Mathematics Teacher, H.S., Mechanicsville, Ia., Head of Mathematics Department, H.S., Pierre, S. D.; Assistant Professor of Mathematics and Critic Teacher, Chadron State Teachers College, Chadron, Nebr.; Head of Mathematics Department, University School, Indiana University, 1942–, and Assistant to the Dean, School of Education, Indiana University, 1950–. Member, CASMT (President, 1951–52); AAUP; NEA; Ind. St. Tchrs. Assn. (Past chmn., Math. Sec.); Phi Delta Kappa (Past Treas. of Alpha Chap.). Author of articles on the teaching of mathematics in THE MATHEMATICS TEACHER, *School Science and Mathematics, School and Society* and *Phi Delta Kappan.*

ROUDEBUSH, ELIZABETH JEAN, Director of Mathematics from Kindergarten through Grade Twelve for Seattle Public Schools, Seattle, Washington.

Graduate of State College of Washington, Pullman, Wash.; A.M., Teachers College, Columbia University; summer sessions at University of Washington, University of California at Los Angeles, University of Southern California.

Activities in NCTM include Member of

Membership Committee, and Discussion Group Leader at 1950 Annual Meeting.

Teacher, Mathematics, Roosevelt H.S., Seattle, Wash.; Mathematics Department Head, Edison Technical School, Seattle, Wash.; Director of Mathematics, Seattle 1949–. During World War II served in the WAVES as Women's Reserve Representative at U. S. Naval Hospital in Chelsea, Mass. Member of ASCD; Wash. Educ. Assn. (former member of State Com. for Public Relations); NEA; Delta Kappa Gamma (Past Vice-president for local chapter); Pi Lambda Theta (Past President of local alumnae chapter); helped to organize the West. Wash. Council of Tchrs. of Math. Publications include *Laboratory Geometry;* "An Arithmetic Bulletin for Parents" in THE MATHEMATICS TEACHER, May 1951.

ELIZABETH JEAN ROUDEBUSH

President's Page

(Continued from page 93)

Hildebrandt, Northwestern University, Evanston, Illinois.

The Policy Committee is sponsored by the American Mathematical Society, the Mathematical Association of America, the American Institute of Statistics, the Association of Symbolic Logic, and the National Council. The Committee has been organized to deal with mathematical questions that are not of direct concern to any one organization and to co-ordinate activities of the various organizations. J. R. Kline, University of Pennsylvania, Philadelphia is chairman of the Policy Committee.

Educational Advisory Committee to Science Service—Veryl Schult, Public Schools, Washington, D. C.

This Committee has been organized to provide for Science Service counsel representative of educational groups and related organizations in the conduct of the Science Clubs of America program, including the National Science Talent Search and the National Science Fair. Watson Davis, Washington, D. C., is Director of Science Service.

Upon invitation of Kenneth Brown, Specialist for Mathematics in the U. S. Office of Education, the Board of Directors voted to ask the Executive Committee of the National Council to assume the responsibility for working out means of co-operation between our organization and the Office of Education. Co-operatively planned activities, jointly planned and carried out by National Council members and the Office of Education, under the leadership of the mathematics specialist in the Office give promise of being among the most effective of our efforts to work with other professional groups.

Suggestions on additional possibilities for co-operation and reports on successful programs of co-operation on a local or national scale will be greatly appreciated.

JOHN R. MAYOR, *President*

HAVE YOU SEEN?

In *The Journal of Engineering Education,* November 1952
- "Youth and Engineering" by General Dwight D. Eisenhower
- "Engineers" by The Honorable Herbert Hoover
- "Minimum Mathematical Requirements for Engineering College Admission and Factors which Affect this Problem"—A Panel Discussion
- "Teaching with Color" by William C. Krathwohl
- "The Secondary School as a Source of Candiates for Engineering Training" by Harry A. Jager
- "A Plan for Training Women in Engineering" by Fred C. Morris

In *Mathematics Magazine,* September–October 1952
- "A Reflective Approach in the Teaching of Mathematics" by Charles A. Johnson
- "Numbers Game"—Reprinted from *Time* Magazine, July 14, 1952

Affiliated Group Activities

By Donovan A. Johnson
Committee on Affiliated Groups

Newsletters for Mathematics Teachers

With the development of modern means of reproduction it has become very convenient to duplicate material inexpensively and quickly. This convenience has given organizations such as Affiliated Groups an effective means of building their organizations and of being of service to its members. Reports, instructional materials, news items, and program announcements are distributed to group members and others by means of newsletters or bulletins. Some of the publications such as the *New Jersey Mathematics Teacher*, the *Bulletin of the Kansas Association of Teachers of Mathematics*, and the *California Mathematics Council Bulletin* have been issued for many years in printed form. Most of the other publications of state or local organizations are mimeographed. This method of mimeographing has a number of distinct advantages. The mimeographed form is very inexpensive to produce. The cost is usually confined to the cost of stencils, paper, and postage since members of the group usually assume responsibility for cutting the stencils, schools permit the use of duplicating equipment, and students assist in such clerical duties as assembling, addressing, and stamping. If 200 or more copies are mailed, a second class permit will reduce the cost of postage. Another real advantage of the mimeographed form is that teachers are much less reticent to write a report if the material is to be published in informal mimeographed form with limited circulation. Thus, it is possible to have many persons participating in the collecting and writing of copy as well as in the actual reproduction and distribution of the material. Publication by mimeographing makes it possible to complete the production in a very short time. There are no delays in waiting for galley proofs, cuts, or printing schedules. Mimeographing also furnishes a versatile medium since drawings, headings, or unusual symbols may be readily cut on the stencils.

Although the size and makeup of newsletters vary greatly, they frequently contain material such as the following:

- Announcements of conferences and meetings for mathematics teachers.
- Organizational information such as memberships, officers, committees, constitutions, meeting minutes, reports and elections.
- Articles on mathematics teaching.
- News notes about local teachers and schools.
- Summaries of talks presented at conferences.
- Application forms for membership in the state organization and the National Council.
- Editorials.
- Greetings from the president.
- Announcement of materials of instruction.
- Unusual sidelights, problems, recreations, quotations of a mathematical nature.
- Reports of current teaching practices.
- Committee reports.
- Book reports.
- Projects for the mathematics laboratory.
- Announcements of workshops and summer school courses.

A newsletter of somewhat different nature is the *O. U. Math Letter* sponsored by the University of Oklahoma chapter of Pi Mu Epsilon. This is a publication directed toward the high school mathematics student. Its purpose is to stimulate interest in mathematics in high school as well as in college. It presents unusual topics in mathematics such as the binary systems and angle trisection, unusual problems and puzzles and news notes about courses, lectures, scholarships, and research at the University of Oklahoma. Students are invited to submit solutions to the problems. The correct solution and the names of the students sending in correct solutions are listed in subsequent issues. Since only one letter is sent to each *senior high school teacher* requesting it, per-

mission is granted to schools to reproduce any material published in the letter.

Although this letter is intended for high school students in Oklahoma it will be sent to *senior high school teachers* outside the state of Oklahoma upon request. The requests should be sent to Professor Richard Andree, Department of Mathematics, University of Oklahoma, Norman, Oklahoma.

A circulation increase from zero to 2,500 copies to mathematics teachers in less than one year is indicative of the success of a newsletter of this type. It would seem that Affiliated Groups in other states could sponsor similar publications in their area. Perhaps another way to fill this need for materials to be used by high school students would be to incorporate material of this nature in the current newsletter of the group.

A list of the publications of Affiliated Groups was announced in the April, 1952 issue of THE MATHEMATICS TEACHER. Although most of the content of these state newsletters are of interest to the state group, many of them are well worth the usual one dollar yearly membership fee.

Affiliated Groups who are publishing newsletters have usually found them to be a highly successful means of stimulating participation in their activities. The selection of a capable publication staff or committee will ensure a newsletter that will be an important factor in the growth and development of your group. Try it!

Other Activities

In addition to newsletters, Affiliated Groups have found committee work on a variety of projects to be highly stimulating. One of the most common projects is the planning of local conferences and workshops. Another is work on the mathematics curriculum. Several state curriculum guides have recently been published, for example, Iowa, Wisconsin, Pennsylvania and Minnesota. The work on these curriculum revisions has often been closely coordinated with activities of the state group. In addition to curriculum study, research is being recognized as being of increasing importance in education even as in industry. The Wisconsin Mathematics Council has recognized this important area by organizing a research committee which is active in sponsoring studies and in gathering materials for secondary and elementary school teachers.

Thus, there are a variety of worthwhile projects which will provide real stimulation for professional growth through the activities of your Affiliated Group.

The First Virginia Institute for Mathematics Teachers will be held at the **University of Virginia, Charlottesville, Virginia, July 27–August 7, 1953** and will follow the pattern so successfully developed by Professor W. W. Rankin at other universities. Principal features include: (1) Morning and evening lectures on applications of mathematics in industry, science and government by persons currently engaged in these fields; (2) Daily study groups throughout the two weeks of the Institute on teaching general mathematics, laboratory approach to the study of mathematics, outdoor mathematics, aids in the study of algebra and geometry, current uses of mathematics to the high school teacher, how to collect, organize and interpret data, mathematics for the gifted pupil and the history, philosophy and psychology of mathematics; (3) Demonstration teaching with a group of children taught daily; and (4) Various social events. Two semester hours may be earned by those who elect to enroll for credit. Persons wishing a copy of the completed program or any other information about the Institute should write to Professor Francis G. Lankford, Jr., 1-B West Range, University of Virginia, Charlottesville, Virginia.

APPLICATIONS

Edited by Sheldon S. Myers
Department of Education, Ohio State University, Columbus, Ohio

Ar. 26 Gr. 6–9 *Speed of Travel in Covered Wagon Days*

Here is a very interesting guide sheet used by Dr. Charles Weidemann, April 12, 1937 at the Ohio State University School.

In the days between 1840 and 1860 many thousands of people traveled from 1500 to 2500 miles to the Pacific Coast from such points as St. Joseph, Missouri, Omaha, Nebraska, and Conneaut, Ohio. They went in search of gold, new soils for farming, and adventure in a new country. Their homes were built in sturdy wagons covered with a canopy of weather-proof canvas and thus received the name "covered wagons." These were drawn by 2 to 6 oxen. Compute the average number of hours of travel per day if the number of hours of travel for each of thirty days was: 6, 11, 8, 9, 14, 4, 10, 12, 7, 8, 12, 3, 9, 14, 9, 9, 5, 13, 6, 8, 11, 11, 11, 7 9, 9, 6, 12, 7, 9.

A brief story of part of such a trip was taken from the diary of Mrs. Frances Sawyer. The trip began on May 8, 1852 from St. Joseph, Missouri. The brief digest of the diary is from May 21 to June 10.

May 21 Mr. Sawyer sick. Rain. Distance 8 miles.

22 Mr. Sawyer better. Gathered prairie peas for pickles. Good roads. Distance 26 miles.

23 Sabbath. Camped 2:00 P.M. with friends. Daughter in bloomer costume of pants, short skirt and red top boots, very appropriate for trip. Distance 16 miles.

24 Warm. Trip monotonous. Good roads. Guard mules from stealing by Indians. Distance 30 miles.

25 Platte River, wide and shallow, water is muddy and warm. Indians beg for food, are ugly, friendly and peaceable. Distance 25 miles.

26 60 to 70 Pawnees, hunting Buffalo, had a fight with 13 Sioux. Two Pawnees scalped—a horrible sight. Large camp of emigrants, good grass, many mosquitoes. Distance 25 miles.

27 Slept well. Picked prairie flowers. Air full of fragrance. Distance 25 miles.

28 Same old monotonous prairie. Seems endless. Distance 30 miles.

29 Fort Kearney. Wrote letters home. Fort neatly kept. We registered. Distance 15 miles.

30 Sabbath. Crossed Platte River, 10 miles west of Kearney. Quicksand. Mules swam some of the way. Wagon watertight so nothing was spoiled. Camped on west bank of river. River one mile wide. Total distance for day ——— miles.

31 Heat and dust. Grass poor. Distance 28 miles.

June 1 Three deaths of cholera. Cold alkali water from shallow wells causes sickness. Distance 28 miles.

2 Buffalo regions. Only fuel is buffalo chips which make a hot fire. Shawnee springs: clear, cold, water is fine. Sweet alkali water is bad. Distance 25 miles.

3 Rain, thunder and lightning last night —cool today. Good water. Distance 25 miles.

4 Many newly made graves from cholera. Distance 25 miles.

5 One of our party sick from cholera. Didn't travel today even though doctors advised everyone to travel.

6 Sabbath. Sick man better. Others ill. Ox teams ill. More rain. Cooler today. Distance 22 miles.

7 Rain, cool and windy. Mr. Sawyer ill. Distance 25 miles.

8 Mr. Sawyer better. Mormons from Salt Lake report better conditions to the west. Castle ruins are large stones on a hill—the work of nature. Distance 25 miles.

9 Many weeks of travel ahead. Emigrants are healthier. Chimney Rock and Courthouse Rock on opposite sides of the Platte. Distance 31 miles.

10 Nooned at Scott's Bluff, named after a man who starved to death near them. Distance 24 miles.

A table is then provided for summariz-

ing the daily travel by tallying the number of times the emigrants: did not travel, went 8 miles, 11 miles, 15 miles, 16 miles, 22 miles, 24 miles, 25 miles, 26 miles, 28 miles, 30 miles, and 31 miles. A column is provided for indicating the total miles traveled at each distance. A grand total is then made of the number of days and the total distance over this period of time. Questions such as the following were then asked:

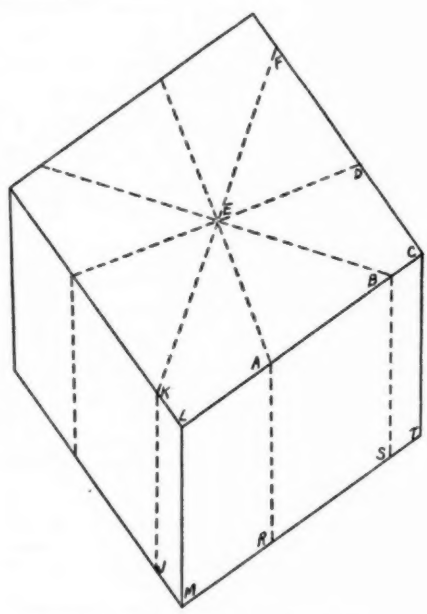
1. How many days did they travel in our summary?
2. What was the total of miles traveled?
3. What was the average miles traveled per day? Hours per day?
4. What was the average miles per hour by oxen or mule team?

Solution of P.G. 10 Gr. 10–11 *Cake Icing*, an Application Submitted by Kenneth Swallow

Briefly restated, the problem involves finding a method for cutting an iced cake, which is in the form of a rectangular solid with square bases, so that each piece has an equal volume of cake and an equal area of icing on the surface. The cake is iced on the top and sides. The solution, indicated by the drawing below, involves the simple operation of cutting from the center E to the edges at K, A, B, D, F, etc. so that $KL+LA=AB=BC+CD=DF$, etc. The proof that this solution is correct depends upon the use of two theorems—one from plane geometry and one from solid geometry. From plane geometry it can be shown that all of the triangles in the upper face have equal areas because they have equal bases and equal altitudes. For the figures that overlap the corners, it can be shown that they are composed of two triangles, the sum of whose areas equal the areas of the other triangles. Thus all the pieces have an equal amount of icing on the top. It can easily be shown that all the rectangles of the sides are equal in area. For the figures on the sides which overlap the corners, it can be shown that they are composed of two rectangles, the sum of whose areas equal the areas of the other rectangles. Thus all the pieces have an equal amount of icing on their sides, consequently making the total amounts of icing on each piece equal.

From solid geometry a special case of the "principle of Cavalieri" is involved.[1] This principle states that two solids of equal altitudes have the same volumes, if plane cross sections at equal height have the same area. It is very easy to show that this condition holds for all the pieces of cake and therefore they have the same volume.

This procedure of cutting a cake requires the division of the perimeter into a number of equal lengths, which is no more difficult to approximate than the usual method of cutting the cake. Regardless of the practicality of the solution, the real value of the problem lies in the combined use of principles from plane and solid geometry.



Solution of the cake-slicing problem.

[1] First extensively used in "Geometria Indivisilibus Continuorum" (1635) by Bonaventura Cavalieri, professor at the University of Bologna.

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by

JOHN A. BROWN
Wisconsin High School
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and

HOUSTON T. KARNES
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Baton Rouge, Louisiana

THE BULLETIN BOARD DEMONSTRATION

The bulletin board is a dynamic way of using visual aids. It is the focus point for the display of material pertinent to a given unit. Many devices can be used to make the bulletin board interesting and attractive. Some suggestions will be found in the following lists:

I. The Physical Aspects of the Board
   A. Unity
      All displays should be planned around a central theme. The *caption* is a very important part of any bulletin board. It should be one around which all items are centered. By this means you can capture the attention with such force that interest will be stimulated.
   B. Clearness and Simplicity
      Usually script is needed with pictures. It should be brief and to the point and printed large enough to be read at a distance.
   C. Color
      The artistic use of color can add much to attractiveness of displays. Remember, you do want contrast. It is wise to let one color dominate the lettering and use a second color to emphasize important points. The color of the bulletin boards themselves is important. Usually they should fade into the background giving sharpness to the items displayed.
   D. Board Display
      There are two ways to display the material on the board:
      1. The *formal* display which is a symmetrical arrangement of the various parts.
      2. The *informal* display is much more flexible and is sometimes more desirable. In this arrangement your eyes must be satisfied with the placement of the various items of your display.

II. The Purposes of the Bulletin Board
   A. General
      1. To furnish aid in achieving certain objectives.
      2. To display material.
      3. To stimulate interest.
      4. To promote an event.
   B. Specific
      1. The Bulletin Board which teaches
         An enlarged chart can be used here. It is one of the most useful forms of teaching aids. There are innumerable situations in which the object to be explained is too small for group instruction. This type of aid has two advantages:
         a. It permits showing of details.
         b. It enables the instructor to teach groups as well as individuals.
         An enlarged "Our Number System" is an illustration of this kind of board.
      2. The Bulletin Board which motivates
         A skillfully arranged bulletin board will stimulate curiosity and serve as an introduction to a particular topic or problem. This type of board is a growing kind—as the newspaper clipping display.
         Displaying of pupils work will motivate them to do their best.
      3. The Bulletin Board which develops a concept
         A chart showing a square inch, a square foot, an inch and a foot are a necessity in any mathematics class. The students can visualize such concepts as a square inch.
      4. The Bulletin Board as an approach to safety
         Safety is of such importance that it should be continually emphasized through the bulletin board and ample space should be provided.
      5. The Bulletin Board which develops vocabulary
         As the mathematical charts of polygons, lines, and angles.
      6. The Bulletin Board which is used as a basis for conversation
         The airplane display as described in the demonstration.
      7. The Bulletin Board which is used for remedial work
         A graph showing the strength and weaknesses in a particular field. This is the starting point for remedial work.

8. The Bulletin Board which inspires
   Men of the Hour
   Good Brotherhood Week
   Pictures which depict the theme of the month.
9. The Bulletin Board for the entire school
   The Bulletin Board may be used to display notices, announcements, school news and information of general interest to all. A school project can also be motivated through the use of this bulletin board.

III. Outcomes and Attitudes developed by preparing the bulletin board
   A. The students learn to work together.
   B. They learn to respect each others opinion.
   C. Through the display of written material for meritorious accomplishment the student is stimulated to do his best work.
   D. They learn to arrange and organize material.

IV. Bibliography of magazine articles
   (All text books on visual education have chapters which deal with bulletin boards.)

Allen, William, "Audio-Visual Materials." *N.E.A. Journal*, XLI (January 1952), 49.

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Civic Training, "Bulletin Boards as Effective Teaching Devices." *American Education Press* (Columbus, Ohio), Vol. XX, No. 11 (November 26–30, 1951)

Fermer, Dolla, "Advertising in the Elementary School Science Class." *Elementary School Journal*, XLI (June 1941), 756–59.

Kaplin, H. H., "Educational Significance of the Bulletin Board." *Peabody Journal of Education*, XXIII (January, 1946) 241–42.

Ohio State University,—"How To Keep Your Bulletin Board Alive." A 32 frame, color filmstrip produced by *Teachers Aids Laboratory, Ohio State University*, Columbus, Ohio.

The filmstrip and a six page bulletin are available from the university.

Starr, G. G. "Teaching by the Bulletin Board." *School Executive*, LXII (September 1942), 26–27.

Stolper, B. J. *The Bulletin Board as a Teaching Device* (Rev. ed.). New York: Teachers College, Columbia University, 1946. 15 pp. (An excellent reference).

Myrtle Lawler
Waukesha, Wisconsin

Mathematics at Hammond Technical Vocational High School

Mathematics at Hammond Technical Vocational High School is in three main divisions: boys shop mathematics, girls shop mathematics, and college preparatory.

The college preparatory mathematics classes are the same as those in other high schools; they are attended by both boys and girls, as preparation for college or technical institute entrance or on recommendation of shop instructors as training for shopwork. Algebra is sometimes recommended for advanced students in electric shop, geometry for some drafting students and for advanced metal layout.

Most of the students enrolled, however, are vocational students who do not expect to continue any formal training after graduation. For them mathematics is a "related subject"; that is, it is related to the particular trades for which they are training. Since Hammond Tech operates with Smith-Hughes vocational funds, mathematics training must follow rather closely the vocational development of each student.

Boys study shop mathematics five semesters. There are no study periods; students are assigned to class or shop eight periods a day. They normally earn 8 work units per semester (one each class period) or 64 for graduation. Of the 64 necessary, five are in mathematics.

Work in shop mathematics is on an individual instruction basis. For that reason, it is impossible to set up an exact amount of work to be covered each semester. The average student spends about two and one-half semesters on basic shop mathematics and the other two and one-half on advanced mathematics relating directly to his chosen trade.

The basic shop mathematics covered by students in all shops includes these units:

1. Fundamentals
   review and improvement of skill in the use of fundamentals
2. Measurement-ruler
   machinist's scale
   protractor
   drafting scale
   micrometer

vernier caliper
3. Fundamental operations applied to units of measurement
4. Computing areas, volumes, and perimeters of common forms
5. Evaluation of formulas
6. Square root and its application

There is almost continuous review on units previously covered. Students are expected to reach and maintain through their mathematics courses an accuracy of at least 80% on all types of calculation covered in the basic units. Since the type of problem covered in these units occurs frequently in the student's shop work, he usually understands the need for a high percentage of accuracy.

The last half of a student's shop mathematics work covers problems associated with his chosen shop. The work is selected by a study of apprentice programs, analyses of the trades, and examination of text books in the field. Much of the instructional material must be prepared by the instructors when such material does not already exist.

The general aim of mathematics in the girls' department at the Technical Vocational High School is to be useful to the student both in classes and shops at school and in her work outside of school. The mathematics courses required are Mathematics I in 9B, Mathematics IV in 10A, and Mathematics V in 11B.

The content of the mathematics courses is related to the study areas available to the girls: Home economics and commercial. General need for some knowledge of cooking and sewing makes it advisable for the content of Mathematics I to be devoted to problems of this nature. Mathematics IV includes units dealing with consumer buying, banking, personal finance, travel and transportation, insurance, public utilities, and taxes. Mathematics V provides problems of a more advanced nature arising in the operation of a typical business. In all mathematics courses offered emphasis is placed on development of fundamental arithmetical operations.

Hammond Technical Vocational High School must take care of part time students who attend sometimes as little as one day a week. This requires the complete use of individual instruction and necessitates careful organization of course content. An extensive file of testing materials is maintained and used for frequent checking on progress. The objective of all Tech mathematics courses is to bring each student to the level of his ability to understand and do. Only when a student is learning at his own capacity is he considered to be doing satisfactory work.

Gerald Kackley
Hammond Technical High School
Hammond, Indiana

Solid Analytic Geometry for Seniors

At Wisconsin High School the third year of sequential mathematics, offered in the eleventh grade, consists of twenty-four weeks of intermediate or "advanced" algebra and twelve weeks of solid geometry. The fourth year course is called elementary mathematical analysis and includes topics from college algebra, trigonometry, analytic geometry, and a brief introduction to the calculus.

In 1951–52 for the first time in the elementary mathematical analysis course, two weeks were given to the analytic geometry of the plane. Following this work the analytic geometry of the circle and the sphere were considered simultaneously. This experiment was carried on in part because less than a semester is given to solid geometry, in the synthetic treatment of the eleventh grade, but more in order to provide an introduction to solid analytic geometry, and to teach the students the easy extension of some of the ideas of plane analytic geometry to topics in solid analytic geometry.

Solid geometry topics covered were: a coordinate system in space, distance between two points, direction components of a line, angle between two lines, equation

of a plane and equations of a line, families of planes, equations of a plane through three points, intersections of planes, equation of a sphere, sphere through four points, tangent plane to a sphere, length of tangent line from a point to a sphere, and the radical plane of two spheres.

The students did not find this work more difficult than other topics in the course and on the final examination the class as a whole did better on questions selected from this unit than on questions from many of the others. While time given to this work had to come from other topics the omissions seem justified because of the opportunity provided for additional work in applications of topics from algebra, such as determinants, and from trigonometry; better understanding of the nature of a co-ordinate system; possible generalizations; additional experience with planes and spheres; demonstration of further advantages of the analytic approach in geometry.

Certainly the experience of last year demands that these topics be included in our course this year, and provides a basis for recommending these topics for senior students, to other teachers who may not have included them in their courses.

JOHN R. MAYOR
Wisconsin High School
Madison, Wisconsin

MATHEMATICS TEACHERS HELP STUDENTS READ BETTER

This semester, Cleveland high school teachers are engaging in an intensified effort to help students improve their reading in six subjects including: English, Mathematics, Science, Social Studies, Home Economics and Industrial Arts. In each subject field a committee of teachers has prepared a list of suggestions for improving the reading of students studying the subject. All teachers of these subjects in the junior and senior high schools are following these suggestions this semester.

Some of the suggestions for improving the reading of students in mathematics classes follow:

1. Build vocabulary understandings.
   a. Write new or difficult words on the board, pronounce them, explain their meaning, then have the pupils pronounce them and use them in sentences. Include technical words such as: numerator, parallel, perpendicular, bisect, isosceles, and exponent.
   b. Study new words in general context, then apply them to specific mathematical situation.
   c. Set up a vocabulary list for each grade and hold pupils responsible for the meaning, pronunciation, and spelling of each word.
   d. Make mathematical symbols meaningful.
2. Don't ask poor readers to sight-read orally.
   a. Each pupil should read silently first, then one may read orally. This will help to diagnose pupils' reading disabilities.
   b. Don't always expect oral reader to tell point of what he has read, for his chief attention may have been on pronunciation.
   c. Teach pupils to slow their reading rate on difficult material and to read it more than once so that the meaning is accurately comprehended.
   d. Teachers should set a good example when reading or speaking to the class by doing it in a meaningful way.
   e. Find something to praise in every sincere effort.
3. Make all reading as purposeful as possible.
   a. Pupils learn best from reading when they recognize the usefulness of what is being read.
   b. Encourage pupils to read the explanatory material and the examples in the text as an aid in solving the problems to be assigned.
   c. Insist that pupils both speak and write in grammatically correct sentences.
4. Set up exercises which develop special skills.
   a. Summarizing rules and grouping formulas are useful review techniques.
   b. Have each pupil write the most important fact he remembers from the preparation of the lesson as a test of his ability to comprehend and generalize.

From the *Newsletter* of the Ohio Council of Teachers of Mathematics prepared by a Committee of Mathematics Teachers in Cleveland

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

68. Inscribing a Square in a Triangle

In Durell's *Modern Geometry* we find a surprisingly simple method of inscribing a square in a given triangle, based on the concept of homothetic[1] figures. The method supplements two of the more conventional approaches to the problem. We are submitting three methods of inscribing a square in a triangle. No doubt there are more. Perhaps readers will provide additional approaches to the problem. We restrict our proofs to the acute triangle case.

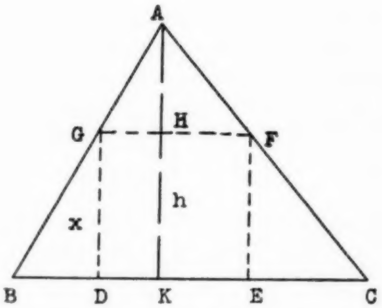

FIG. 1

FIRST METHOD: (Fig. 1). Let x be a side of the square and $AK=h$ be the altitude from A to BC. Let $BC=a$. By similar triangles: $BC/GF=AK/AH$. Substitute $BC=a$, $GF=x$, $AK=h$, $AH=h-x$. Then $a/x=h/h-x$, $xh=ah-ax$, $x=ah/a+h$, or $a+h/a=h/x$. Hence to find x construct the fourth proportional to $a+h$, a, and h, all of which are known.

SECOND METHOD: (Fig. 2). On altitude AD construct the square $ADFE$.

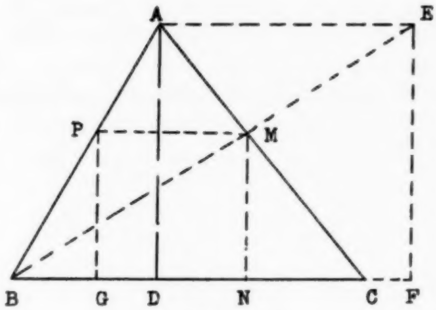

FIG. 2

Draw BE intersecting AC at M. From M draw MN parallel to EF and MP parallel to AE. Draw $PG\perp BF$. Then $PMNG$ is the required square.

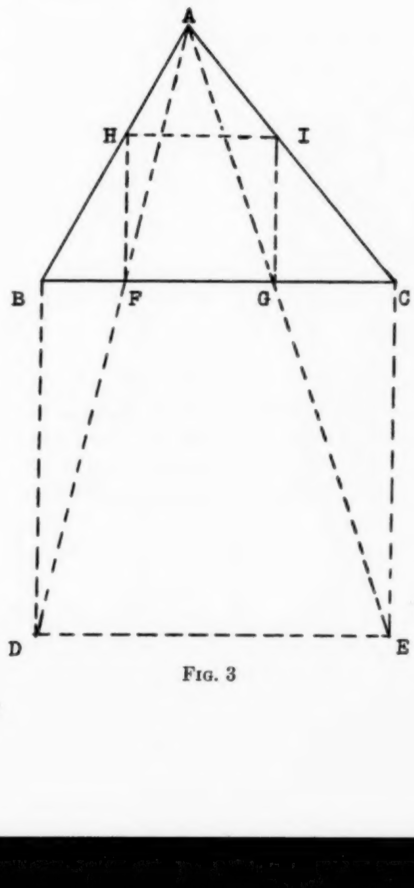

FIG. 3

[1] Two geometric figures which are similar and in addition have sides parallel are said to be *homothetic* or *homothetically situated.*

Proof: By similar triangles, $PM/AE = BM/BE = MN/EF$. But $AE = EF$, so $PM = MN$. $PG \| MN$, $PM \| AE \| BF$. Hence *PMNG is a square.*

THIRD METHOD: (Durell's) (Fig. 3). With BC as a side, construct square $BCED$. Draw AD and AE intersecting BC at F and G respectively. Construct perpendiculars to BC from F and G intersecting AB and AC at H and I respectively. Then $HIGF$ is the required square.

Proof: $BCDE$ and $HIFG$ are homothetically situated with A the center of similitude.

MARTIN HIRSCH
Junior High 227
Brooklyn, New York

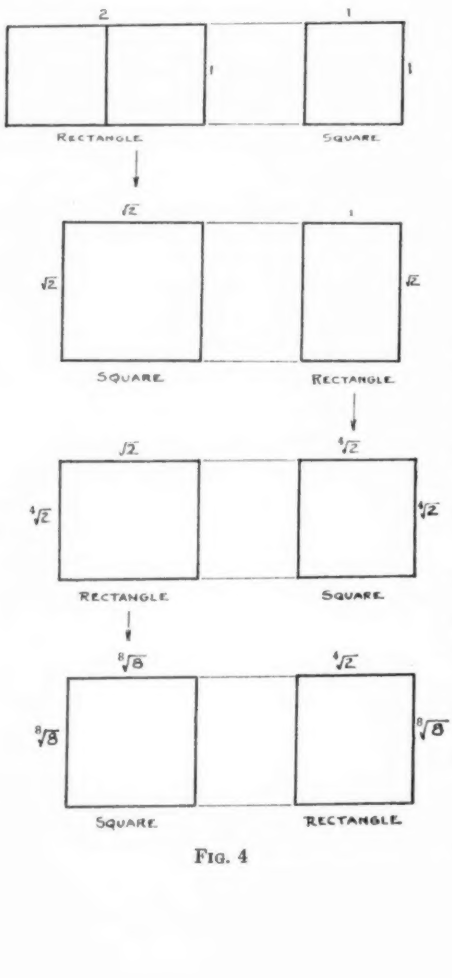

FIG. 4

69. Duplication of the Cube

If two cubes of unit edge be placed together as shown in Figure 4, a parallelepiped results. If the rectangular face be converted to a square, the second parallelepiped will have the same volume and its three dimensions are more nearly equal.

If in the second parallelepiped the rectangular side be converted to a square, the third parallelepiped will have the same volume as before and its three dimensions $\sqrt{2}$, $\sqrt{2}$, 1 will be more nearly equal.

If this process be continued indefinitely the figure approaches as a limit a cube twice the size of the original cube.

While this procedure is not particularly profound, the proof of it calls for some interesting algebra, but not more than can be followed by a student familiar with the laws of exponents, the infinite geometric series, accompanied by a short discussion of induction.

As shown on the diagram the sides of the successive squares can be gotten by finding the mean proportional between the sides of the successive rectangles. The sides of the squares are in succession

$$2^0,\ 2^{1/2},\ 2^{1/4},\ 2^{3/8},\ 2^{5/16},\ 2^{11/32},\ 2^{21/64}\ \ldots$$

If each term of this sequence be divided by the term preceding it, we have: $2^{1/2}$, $2^{-(1/4)}$, $2^{1/8}$, $2^{-(1/16)}$, $2^{(1/32)}$, $2^{-(1/64)}$, $\ldots$ Hence the n^{th} term of the original series will be the product of $(n-1)$ terms of the 2^{nd} series. This product is

$$2^{(1/2-1/4+1/8-1/16+1/32-1/64+\cdots(-1)^n/2^{n-1})}$$
$$= 2^{1/3[1+(-1)^n/2^{n-1}]}$$

This can be shown, by mathematical induction, to be the general expression for the side of the successive squares.

If the geometric procedure is continued an "infinite number" of times, the side of the resulting square approaches as its limit

$$2^{1/3} = \sqrt[3]{2}.$$

Hence the greater the number of steps to

which we carry our construction the more nearly the cube has been duplicated.

HUGH H. MCCLELLAND
Episcopal Academy
Philadelphia 31, Penn.

70. *The "1-2-3" Proposition*

THEOREM 1. *The shortest of the lines terminated by the rectangular axes that can be drawn through a given point (in a plane) is the one with the property that the cube of the distance between the foot of the perpendicular from the point to the X-axis and the x-intercept of the line is equal to the product of the first power of the abscissa and the second power of the ordinate of the given point.*

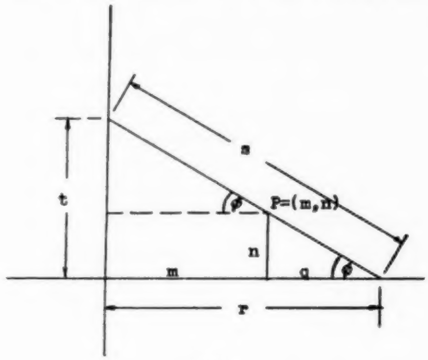

FIG. 5

For, if s is the length of the line (Fig. 5) and $P=(m, n)$ is the given point then $s=m \sec \phi+n \csc \phi$ and $ds/dt=m \sec \phi \tan \phi-n \csc \phi \cot \phi$. Equating $ds/dt=0$, we derive

$$\frac{\sec \phi \tan \phi}{\csc \phi \cot \phi}=\frac{n}{m},$$

or finally

$$\tan^3 \phi=n/m. \tag{1}$$

But $\tan \phi=n/q$ and hence

$$\tan^3 \phi=n^3/q^3. \tag{2}$$

Thus $n^3/q^3=n/m$, or $mn^2=q^3$, or $n^1m^2=q^3$.

The exponents, 1-2-3, in the last expression justify our title for the proposition.

THEOREM 2. *The locus of all points for which the length of the "shortest line" (in the sense of Theorem 1) is a given constant, s, is the hypocycloid of four cusps, or astroid.*

For $s^2=r^2+t^2$, and, since $r=m+q$ and $t=r(n/q)$, $s^2=(m+q)^2+(rn/q)^2$ which, with the substitution $q^3=mn^2$ to impose the "shortest line" condition, and with the taking of square roots becomes

$$s=(m^{2/3}+n^{2/3})^{3/2}. \tag{3}$$

Substituting (x, y) for (m, n) we have finally, as stated in the theorem, that the desired curve is

$$x^{2/3}+y^{2/3}=s^{2/3}. \tag{4}$$

PROBLEM 1. *Find the length of the "shortest" tangent terminated by its axes which can be drawn to an ellipse.*

Let s be this length and $b^2x^2+a^2y^2=a^2b^2$ be the ellipse (Fig. 6).

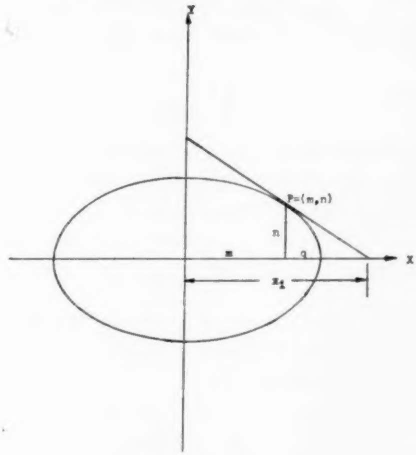

FIG. 6

If $P=(m, n)$ is the point of tangency, the equation of the tangent is $b^2mx+a^2ny=a^2b^2$, and its x-intercept is $x_i=a^2/m$. If this is the shortest line through P, then, by Theorem 1, $x_i=m+q$ where $mn^2=q^3$, or $x_i=m^{1/3}(m^{2/3}+n^{2/3})$.

Thus

$$\frac{a^2}{m}=m^{1/3}(m^{2/3}+n^{2/3}),$$

or

$$a=m^{2/3}(m^{2/3}+n^{2/3})^{1/2}. \tag{5}$$

Similarly

(6) $$b=n^{2/3}(m^{2/3}+n^{2/3})^{1/2}.$$

Adding (5) and (6) we have

(7) $$a+b=(m^{2/3}+n^{2/3})^{3/2}$$

where a and b are the semi-axes of the ellipse.

Finally from (3) we see that this is also the length of the shortest tangent, i.e.

(8) $$s=a+b.$$

PROBLEM 2. *Find the coordinates of the point of tangency of this shortest tangent.*

Dividing (5) by (6) we have $a/b=m^{2/3}/n^{2/3}$, whence

(9) $$m^2=\frac{a^3n^2}{b^3}\cdot$$

Substituting m and n into the equation of the ellipse gives $b^2m^2+a^2n^2=a^2b^2$ or $b^2(a^3n^2/b^3)+a^2n^2=a^2b^2$ from which

(10) $$n=\sqrt{\frac{b^3}{a+b}}\cdot$$

Similar procedures, or substituting this value of n in the equation, or merely considerations of symmetry give

(11) $$m=\sqrt{\frac{a^3}{a+b}}\cdot$$

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Department Editor's Note: The work in Problem 1 given here merely shows the derived length to be the length of that tangent which is also the shortest line through its point of tangency. It is also true that this tangent is the shortest of all possible tangents to the quadrant of the ellipse.

The relationships between the line of constant length, s, which slides with its ends on the coordinate axes, the astroid, and a family of ellipses are interesting.

The astroid is the envelope of all possible positions of this line. Students will find it fun to draw a number of these lines and see a curve be "formed" by straight lines.

Further, if any point is taken on this sliding line, the path it traces as the line moves is the ellipse whose semi-axes are the two segments of the line (this is the idea behind the elliptic trammel, a device for drawing ellipses). The astroid is also the envelope of the family of all such ellipses. For definitions of the astroid and the hypocycloid in general together with drawings of the astroid as the envelope of both the lines and the ellipses see Robert C. Yates, *A Handbook of Curves and Their Properties.* (Ann Arbor: J. E. Edwards, 1947).

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71. A Suggestion for Duodecimals

There seems to be a considerable amount of space being given to literature on the use of various number systems other than our well-known decimal system. Particular emphasis is being put on the advantages and disadvantages of the duodecimal number system.

It is unfortunate that the proponents of this idea for helping to appreciate number relationships, have found it convenient to use the letters X and E for *dek* (or *dec*) and *el*, respectively.

The meaning of letters as letters is difficult to by-pass in a student's mind. Simply to overcome the block set up by the tendency to call X by the word x and E by the word e involves considerable effort. Also, X means 10 in Roman numerals and multiply in arithmetic. Why wouldn't it be better to use some regular printers' symbols (ones easily obtained for textbook work) such as ⊗ for *dek* and ∅ for *el?* These symbols have no particular meaning to the student, and can be placed in line with arabic numbers without need for relearning. Since the symbols have no well-known designation for the student, as do X and E, he would be saved the trouble of considerable interference of previous knowledge with new learning; he would not know the ⊗ and ∅ from any previous teaching.

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72. Conditions under which a Wrong Procedure Gives a Correct Answer!

A student, in solving the equations

(a) $$y=x+5$$

(b) $$(x-2)^2+(y-3)^2=16$$

simultaneously, proceeded as follows: He replaced equation (b) by the pair

$$\text{(c)} \qquad (x-2)+(y-3)=4$$

and

$$\text{(d)} \qquad (x-2)+(y-3)=-4.$$

Then, from (a) and (c) he obtained the solution (2, 7), and from (a) and (d) the solution (-2, 3). These are the correct results, since these number pairs satisfy the given equations.

As a matter of interest, we investigate the conditions under which this incorrect procedure yields the correct solutions.

Consider the equations

$$\text{(1)} \qquad y=x+c$$

$$\text{(2)} \qquad (x-a)^2+(y-b)^2=r^2.$$

Then, pair equation (1) with

$$\text{(3)} \qquad (x-a)+(y-b)=r$$

and

$$\text{(4)} \qquad (x-a)+(y-b)=-r.$$

Solving equations (1) and (3) simultaneously yields

$$x=\frac{a+b-c+r}{2}, \qquad y=\frac{a+b+c+r}{2}.$$

Upon substituting these values in (2), we obtain

$$\left(\frac{r-a+b-c}{2}\right)^2=\left(\frac{r+a-b+c}{2}\right)^2=r^2,$$

which when simplified reduces to

$$\text{(5)} \qquad r=\pm(a-b+c).$$

The same result is obtained when we solve simultaneously equations (1) and (4) and then substitute in (2). Thus, it follows that equation (5) expresses the necessary and sufficient condition that the suggested incorrect procedure will yield the correct solution.

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Department Editor's Note: A geometric approach to this problem adds some interest and clarity.

Lines (3) and (4) when written as

$$\text{(6)} \qquad y-\overline{b\pm r}=(-1)(x-a)$$

are seen to be parallel, with an angle of inclination of 135°, and through points (Fig. 3) $F=(a, b+r)$ and $E=(a, b-r)$, ends of the vertical diameter of the circle. Hence the other intersections of (3) and (4) with the circle are points D and G, extremities of the horizontal diameter.

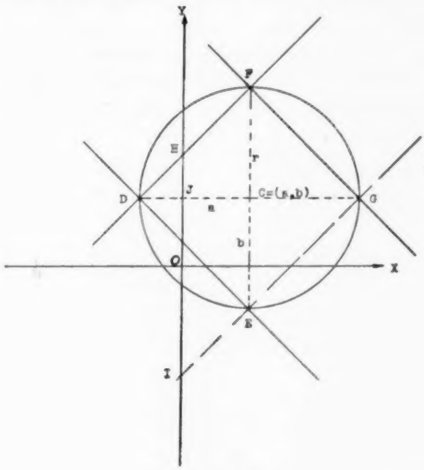

Fig. 3

For the intersection of these lines with line (1) to be the same as line (1)'s intersection with the circle, line (1), which is perpendicular to (3) and (4), must pass through D and F or E and G.

Since DF has a slope of 1, $\triangle DJH$ is isosceles and $JH=DJ=r-a=-(a-r)$; hence the y intercept of DF is $OH=b-(a-r)$. Similarly the y intercept of line EG is $OI=OK=b-(a+r)$.

Finally then, $y=x+c$ will pass through D and F or E and G according as $c=b-(a-r)$ or $c=b-(a+r)$. These conditions on c are readily seen to be equivalent to condition (5) above.

P.S.J.

73. Approximate Square Roots

An interesting article in The Mathematics Teacher of January, 1952[2] discussed the computation of square roots by an averaging method which can be represented by the formula:

$$\text{(1)} \qquad \sqrt{n}=1/2\left(a+\frac{n}{a}\right)$$

[2] Carl N. Shuster, "Approximate Square Roots," The Mathematics Teacher. XLV (Jan., 1952), 17–18.

where a is any positive rational number taken as a first approximation to $\sqrt{n}$.

This method was rightly claimed to be the most rapid and efficient for changing the 3-digit approximations of a slide rule into more accurate approximate roots.

We will describe two other interesting methods for finding approximate roots.

Computation of Square Roots through Continued Fractions: This method is derived as follows:

Consider the quadratic equation $x^2+ax=k$ where k is a positive rational number. Its roots are:

$$(2)\qquad x_1=\frac{-a-\sqrt{a^2+4k}}{2} \quad\text{and}\quad x_2=\frac{-a+\sqrt{a^2+4k}}{2}.$$

Evidently $x_1<0$ and $x_2>0$.

Now, $x^2+ax=k$ can be written $x(x+a)=k$ or,

$$x=\frac{k}{a+x}.$$

But if x in the right member is again replaced by $k/(a+x)$ then we have

$$x=\cfrac{k}{a+\cfrac{k}{a+x}}.$$

Repeating this operation again and again indefinitely, we find:

$$x=\cfrac{k}{a+\cfrac{k}{a+\cfrac{k}{a+\cfrac{k}{a}\cdots}}}.$$

This is a positive number, hence by equating it to x_2 we find:

$$\frac{-a+\sqrt{a^2+4k}}{2}=\cfrac{k}{a+\cfrac{k}{a+\cfrac{k}{a}\cdots}}.$$

Which becomes:

$$(3)\qquad \sqrt{a^2+4k}=a+2\left[\cfrac{k}{a+\cfrac{k}{a+\cfrac{k}{a}\cdots}}\right].$$

Setting

$$(4)\qquad \sqrt{n}=\sqrt{a^2+4k} \quad\text{or}\quad n=a^2+4k.$$

We have evidently as a:

1st approximation to $\sqrt{n}$: $\quad a$

2nd approximation to $\sqrt{n}$: $\quad a+2\cdot\dfrac{k}{a}$

3rd approximation to $\sqrt{n}$: $\quad a+2\left[\cfrac{k}{a+\cfrac{k}{a}}\right]$

and so on.

Each of these rational approximations are called convergents. For instance for $\sqrt{2}$, choosing $a=1$ as the first approximation (then $a^2+4k=2$ gives $k=1/4$), the convergents are:

$$(5)\qquad 1;\ 1\frac{1}{2};\ 1\frac{2}{5};\ 1\frac{5}{12};\ 1\frac{12}{29};\ 1\frac{29}{70};\ 1\frac{70}{169};\ 1\frac{169}{408}\cdots.$$

The continued fractions with positive terms as they are considered above are characterized by the following three important properties:

a. They always converge (approach a limit), which is intuitively to be expected since they represent a real root of a quadratic equation.[3]

b. This convergence is very fast.

c. The errors can be readily estimated.

I tried to find out which of these two methods was the more rapid and I was amazed by the fact that both methods lead to the same rational approximations, with this in favor of the averaging method: it was eliminating a considerable

[3] Formula (3) still holds and the continued fraction converges for $k<0$ provided $a^2+4k\geq 0$.

number of the convergents of the continued fractions. This method produced only the 1st, 2nd, 4th, 8th, 16th, . . . convergents thus making the computation considerably faster.

Calling the 1st, 2nd, 3rd $\cdots$ approximations of the first method $\alpha_1, \alpha_2, \alpha_3, \cdots$, and $\beta_1, \beta_2, \beta_3, \cdots$ the 1st, 2nd, 3rd $\cdots$ approximations of the second method we have found the following:

$$a=\alpha_1=\beta_1;\ \alpha_2=\beta_2;\ \alpha_3=\beta_4;\ \alpha_4=\beta_8;\ \alpha_5=\beta_{16};\ \alpha_6=\beta_{32}, \cdots. \tag{6}$$

And indeed these findings can be easily established mathematically.

In fact with respect to the second or continued fractions method, we saw that when a is the first approximation to $\sqrt{n}$, then the second approximation is:

$$a+\frac{2k}{a}\cdot \tag{7}$$

But from (4) we derive,

$$k=\frac{n-a^2}{4}\cdot \tag{8}$$

Substituting this value of k in (7) we obtain:

$$a+\frac{2k}{a}=a+\frac{2}{a}\left(\frac{n-a^2}{4}\right)=a+\frac{n-a^2}{2a}$$
$$=\frac{1}{2}\left(a+\frac{n}{a}\right)$$

which is just the formula of the first method.

We therefore have proved that if the first approximation was a, the application of both methods give as the second approximation the same result, that is $\alpha_2=\beta_2$. In a similar way it can be proved that $\alpha_3=\beta_4$, $\alpha_4=\beta_8$, and so on.

There are more general approaches to the theory of continued fractions. Perhaps some readers will find it interesting to explore them.[4] The limits for the error resulting in using any convergent of a continued fraction to represent the continued fraction are well established and well known in this theory. They can be easily adapted to the averaging method too since the methods are closely related to each other. However, let's here consider only the uses of the continued fraction procedure in connection with improving slide rule approximations as Shuster did for the averaging method in the article I cited at the beginning. Let us assume that the error in a slide rule approximation is in the neighborhood of 1 in 1000. The following relations then hold:

$$\sqrt{n}\approx a+0.001a \quad \text{or} \quad n=a^2\left(\frac{1001}{1000}\right)^2. \tag{9}$$

Now, if a is the first approximation, the continued fraction procedure gives as a second approximation:

$$a+\frac{2k}{a},$$

where as we know:

$$k=\frac{n-a^2}{4}\cdot$$

Using (9) this becomes:

$$k=\frac{2002001}{(1000)^2}\cdot\frac{a^2}{4}\cdot$$

Substituting this value of k in (7) we find:

$$a+\frac{2k}{a}=\frac{2002001a}{2000000}\cdot \tag{10}$$

Now, the difference between this second approximation (10) and the first $\sqrt{n}\approx 1.001a$ is:

$$\frac{2002001a}{2000000}-\frac{1001a}{1000}=0.0000005a.$$

This means that if the first approximation is determined with an error of $0.001a$, then on the second approximation the error becomes of the order of $0.0000005a$. Hence if we use for a the data given by a slide rule, and continued fractions to find a second approximation, the resulting error

[4] See for example H. S. Hall and S. R. Knight, *Higher Algebra*. (London: Macmillan and Co., 1932). Chapters XXV, XXVII, XXXI, or Chrystal's *Algebra* recently reprinted by the Chelsea Publishing Co.

will be in the neighborhood of $0.0000005a$. This is also the error after the first application of the averaging method since both methods give the same result thus far. In fact, this value of $0.0000005a$ was found by Shuster in his evaluation of the error in the article cited without indicating that his method was nothing else than a variant of continued fractions. However, his method is the simpler and faster. We shall reach this same conclusion after considering the second of our methods.

Computation of Square Roots Using Newton's Method. Once more let n be the number of which the square root is desired. We can write:

$$(11) \qquad x=\sqrt{n} \quad \text{or} \quad x^2-n=0.$$

Let us set:

$$f(x)=x^2-n.$$

Then differentiating:

$$f'(x)=2x.$$

We know that if a is a number in the neighborhood of a root of equation (11), then we have a better approximation for this root x, by Newton's formula:

$$x=a-\frac{f(a)}{f'(a)}=a-\frac{a^2-n}{2a}=\frac{a^2+n}{2a}$$

$$=\frac{1}{2}\left(a+\frac{n}{a}\right).$$

That is once more we find our formula (1).

We will not go into the details of the well known theory of errors in Newton's Method, but it is worthwhile to mention that for a conveniently approximated number a such as the value given by a slide rule, a single application of Newton's Method generally doubles the number of the significant digits. Hence this must be true also for our equivalent first method. That is, if a, the first approximation to n, is a 3-digit number, then the application of the first method will give a second approximation correct to 6 significant digits.

If this last process is extended to the computation of r^{th} roots, we shall obtain the following formula

$$\sqrt[r]{n}\approx\frac{1}{r}\left[(r-1)a+\frac{n}{a^{r-1}}\right]$$

where a is a rational number taken as first approximation.

Although Newton's Method is of a general character, in the particular case of the computation of square roots, it too is related to continued fractions.

We can now state in conclusion that the formula

$$\sqrt{n}\approx\tfrac{1}{2}(a+n/a)$$

which expresses symbolically an old method known by the ancient Greeks, symbolizes and simplifies both continued fractions and Newton's method. And for this reason it is preferable to them.

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The University of Houston will conduct its **Third Mathematics Institute** on **June 23–26, 1953.** Guest consultants will be Dr. Howard F. Fehr, Professor of Mathematics and Chairman of the Department of the Teaching of Mathematics, Teachers College, Columbia University and Associate Editor of The Mathematics Teacher and Dr. Ben Sueltz, Professor of Mathematics and Chairman of the Graduate Division, State University Teachers College, Cortland, New York. In addition to the usual discussion groups, the Institute will feature four laboratories for teachers at various grade levels. The junior and senior high school laboratories will be conducted by Miss Ida May Bernhard, Consultant in Secondary Education of the Texas Education Agency. The laboratory for primary teachers will be conducted by Miss Cecile Foerster, Supervisor of Kindergarten and Primary Grades, Houston Public Schools. Miss Joyce Benbrook, Associate Professor of Elementary Education at the University of Houston will be in charge of a laboratory for teachers of intermediate grades. Many other features of interest to teachers of mathematics at all levels will be highlighted.

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Edited by William L. Schaaf
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The Measurement of Time

"If anyone asked me to define time, I should reply: Do you know what it is that you speak of? If he said Yes, I should say, Very well, let us talk about it. If he said No, I should say, Very well, let us talk about something else." (Attributed to Poinsot by Claude Bernard.)

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. . . **mathematics is good human talk** . . . a language of action and relation . . . useful, well ordered . . . a shield and buckler against verbal confusions . . . It developed to meet urgent human needs . . . it has extended knowledge into unprecedented areas, and the extension goes steadily forward.

—Stuart Chase, in *The Tyranny of Words*, Harcourt Brace and Company.

AIDS TO TEACHING

Edited by

Henry W. Syer
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and

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BOOKLETS

B. 119—Sets and Logic

Mr. Mel Lieberstein, Dahlgren, Va.

Booklet (mimeographed); $8\frac{1}{2}'' \times 11''$; 18 pages; Single copies, \$.65; 10 or more \$.50 each (returnable sample available).

Description: This is a supplement to plane geometry courses, intending to increase logical thinking by introducing formal logic directly into the course. The following topics are discussed: sets, Euler circles, syllogisms, converses, and the slightest introduction to the Boolean algebra of sets.

Appraisal: The style of writing seems to be that which would appeal to high school students and so do most of the examples used. The topics chosen to stand for "logic" are sketchy and of moot importance; other choices might be better. The basic assumption that the teaching of such purely logical ideas will increase the logical thinking of the students is a shaky assumption. It can certainly do no harm, and in the hands of an interested and informed teacher it can be very clarifying of the thought process. Along with other procedures, it certainly can increase the amount of logical thinking done. The typing, drawing and mimeographing are very clear and neat. The price of the 18 pages is all out of proportion to its usefulness. If this material cannot be found in professional journals, nor incorporated into professional courses or regular textbooks, then few teachers will be interested to pay such prices for one idea. There are too many new little bits of information we would like for our work. If it can be sold cheaper, it will be very valuable for its purpose.

B. 120—Bibliography of Recent Air Age Education Textbooks (Cat. No. C31.102:T 31/950)

B. 121—Bibliography on Aviation for Guidance Counsellors (Cat. No. C31.102: Av5/4/950)

Superintendent of Documents, Government Printing Office, Washington 25, D.C.
Booklets; $8'' \times 10''$; Prices below.

Description of B. 120: (50 pages, \$.30) All types of aviation references of interest to children are first classified on ten pages by grade level and subject area. Then the same references are listed alphabetically by authors with excellent annotations.

Description of B. 121: (12 pages, \$.15) Two pages of classified titles are followed by an annotated list by authors of books which describe the vocations associated with aviation.

Appraisal of B. 120 and B. 121: These were compiled in 1950 so should still be of great use for libraries, classroom teachers and parents. There is little connection with mathematics except that students in mathematics classes and interested in mathematics are more apt to be interested in aviation.

B. 122—Courses of Study in Office Machines and Clerical Practice

B. 123—The Teaching of Office Machines

B. 124—Scratch Pad Demonstration

B. 125—Teacher's Guide for Educator Monroe Adding Calculator

Educational Department, Monroe Calculating Machine Co., Orange, N. J.
Booklets; Free

Description of B. 122: ($5\frac{1}{2}''\times 8\frac{1}{2}''$, 11 pages) A description of study in the Newark schools only, telling their topics and methods.

Description of B. 123: ($6''\times 9''$, 9 pages) Two articles on teaching and evaluating instruction in office machines.

Description of B. 124: $5\frac{1}{2}''\times 8\frac{1}{2}''$ (about 25 pages, all alike) There are printed examples of addition, subtraction, multiplication and division with their answers, to be used in connection with the Teachers Guide (B. *125*) in demonstrating the use of the machine.

Description of B. 125: ($8\frac{1}{2}''\times 11''$, 20 pages, one side) In parallel columns this booklet tells how to perform the four fundamental operations on the Monroe Educator Model; one column tells what to say and the other what to do.

Appraisal of B. 122 through B.125: These are specialized booklets which serve the purposes described very well.

B. 126—Truck Walks on Wheels

Mechanix Illustrated, 67 West 44th Street, New York 36, N. Y.

Leaflet; $6''\times 9''$; 2 pages; Free

Description and Appraisal: This reprint shows a method of using four elliptical wheels to increase bearing surface and traction on a new type of truck. It is an unusual application of mathematics and could lead to a very interesting discussion to discover why they work.

EQUIPMENT

E. 121—Fracto-Blox

The Playway Games, 18 Division Street, Sidney, N. Y.

Arithmetic game; $7\frac{1}{2}''\times 7''\times 1\frac{1}{2}''$; \$3.95.

Description: There are 45 pieces of $1\frac{1}{2}''$ dowling with their lengths proportional to the fractions stamped on them; the "1" block is 2 inches long. The distribution of pieces as follows: 1–1; $\frac{1}{2}$–3; $\frac{1}{3}$–4; $\frac{1}{4}$–7; $\frac{1}{6}$–14; and $\frac{1}{8}$–16. Each size is dyed a different color to make recognition easy.

Appraisal: The lengths of the blocks seem to be machined accurately enough for their purpose. The color adds to the attractiveness and the usefulness in distinguishing the fractions. The blocks are a little small for demonstration with very large groups, but are an excellent size for individual work. There are ten "facts" for demonstration which are listed on a mimeographed sheet which accompanies the game. Any teacher will think of other uses as well. Some of the language will certainly be changed; for example, "the larger name-number (denominator) indicates the smaller part." A recent letter says such changes are being made. Certainly this game should be in the school collection.

FILMS

F. 74—Maps Are Fun

Coronet Instructional Films, Coronet Building, Chicago 1, Ill.

Educational Collaboration, Dr. Viola Theman; September, 1946.

B&W (\$40) and Color (\$80); 400 feet; 11 min.

Description: This film introduces the fundamental concepts of map reading such as legend, scale, grid, types of graphs, uses of color, and how to read a map index. Two boys, Ronnie and Dick, take the problem of making a map of a paper route to a cartographer, Mr. Donaldson. He shows them how to work to scale and how to use symbolism and directions in making their simple map. Other facts are brought out as the boys succeed in making a serviceable map and in learning more about maps in general. The three characters, Ronnie, Dick, and Mr. Donaldson, act out their parts while an unseen fourth party provides the verbal descriptions of the story.

Appraisal: Since this film was not constructed specifically for the mathematical curriculum, it is unfair to judge it against objectives set up by the reviewer. However, the film does a fair job of meeting the three additional objectives he has set up.

Assuming that the ratio concept is not new to the junior high student, the objective best met is that of showing the nature, use, and power of symbolism. To use this film efficiently in the classroom the students should be familiar with the ratio concept. The ideas move too quickly for a first understanding. The student should have many different types of maps available at their own desks so that comparisons can be made of the different scales, grids, and symbols. The map symbolism could be shown to be the same kind of symbolism used in mathematics—for example: the letter representing a number of unknown value, the symbols of operation and the number symbols.

This film is a good supplementary tool for all three objectives. The vocabulary and illustrations are excellent for junior high school students. The speed of development of ideas is too rapid for a first study of anything except symbolism although the speed is just right for a supplementary unit on ratio and grid.

The film is interesting, authentic and leads to student projects of map construction.

The technical qualities of the film are excellent in most respects. The interest in the film could have been enhanced by the use of the color copy. (The black and white film was the one used for evaluation.) Also, actual talking by the main characters would have been more realistic to the pupil. The junior high student is often critical of depicted reality—for example, many pupils wondered whether two paper boys would go to all of the work that Ronnie and Dick did just to deliver papers. (Reviewed by PERRY CHAPDELAINE, Mason City, Iowa.)

FILMSTRIPS

FS. 67A, FS. 68A, FS. 153–FS. 156—History of Measures Series

F.S. 67A—Our Number System

F.S. 153—Our Calendar

FS. 154—Telling Time

FS. 68A—Linear Measures

FS. 155—Area Measures

FS. 156—Weight and Volume

Young America Films, 18 East 41st Street, New York 17, N. Y.

35 mm. filmstrips; 6 strips; Black and white; $3.50 each, $16.50 set

Description of FS. 67A: This filmstrip shows how early man used objects such as sticks, pebbles, shells, or marks to show how much or how many. Later pictures and tally marks gradually evolved into symbols for numbers. However, the number symbols of the Greeks, Hebrews, Hindus, and Romans were very different. Early methods of computing consisted of counting objects or using an abacus. The invention of zero as a place holder simplified computation. Other symbols such as $+$, $-$, $\sqrt{}$ were developed to help us use numbers and make our modern world possible.

Description of FS. 153: Time is divided into units many of which are based on cycles found in nature. Days, months, and years are shown to be related to cycles of our solar system. The Babylonians, Greeks, and Romans developed calendar years of 12 months with 29 or 30 days in each month which necessitated frequent corrections. Julius Caesar used the Egyptian year of 365 days to develop a calendar year beginning with March. Our modern calendar was devised in 1582. Calendar dates vary depending upon the event from which they are numbered. Thus, the Christian, Hebrew, and Mohammedan dates do not correspond. The last scenes refer to the possibility of considering

changes and improvements in our present calendar.

Description of FS. 154: After illustrating the need for accurate time, the development of time measurement from the caveman to the present is discussed. After crude shadow measurement, the sun dial added accuracy to time measurement, but the sun dial was useless on cloudy days, so water clocks, hour glasses, and candles were next devised. However, it was not until Galileo discovered the regularity of pendulum swings that accurate mechanical clocks were made. Illustrations of hair spring and balance wheel show how modern watches operate.

Description of FS. 68A: The process of manufacturing automobilies is used to illustrate the need for exact standards and accurate measurement. Early units of measure such as the cubit were based on parts of the human body and hence, varied greatly. Simple standardization of the cubit resulted from using a stick to represent a specific cubit. This made projects such as building pyramids possible. Similarly, other units such as the foot, yard, inch gradually became standardized. The development of the metric system and exact standards and instruments are making improved measurements possible.

Description of FS. 155: This filmstrip begins by contrasting the importance of measurement now and in the past. Area measurement probably began with land measurement in inaccurate units, such as the amount of seed needed to plant it. Other units for measuring land, such as acre and rod became standardized and a number system for computing was devised so that today measurement and computation make modern building and mechanical devices possible.

Description of FS. 156: This filmstrip traces the development of measuring devices from the crude seashell or gourd to the modern standardized units such as liters and pounds. The ancients used seeds, stones, and metal weights or balance scales to measure weight. Standards for measuring weight developed early because of trade, an example, of which was the pound which was standardized by the Greeks. The units of capacity, e.g., the pint, quart, gallon, and bushel were based on sizes of containers, but varied greatly. Governmental action resulted in the standardization of units of measurement. Despite the convenience and accuracy of the metric system, we still use cumbersome, although standardized and accurate, units of measure of the English system.

Appraisal of FS. 153–FS. 156: This series of filmstrips will supply mathematics teachers with valuable supplementary background material. It is material that lends itself well to presentation by filmstrips. The pictures are very good and the captions brief and appropriate. At times the reviewer felt that the treatment of a topic was too brief and incomplete. A good point of emphasis was the dependence of modern civilization on accurate measurement and computation. This series will be usable in the sixth grade or junior high school mathematics classes. These filmstrips combined with pamphlets, charts, and instruments previously reviewed in this section and books such as *How Much and How Many* will make it possible for teachers to build excellent units of instruction.

FS. 157—How to Tell Time: Part I: The Hour and Half-hour

FS. 158—How to Tell Time: Part II: The Minutes

Popular Science Publishing Company, 353 Fourth Avenue, New York 10, N. Y.

B&W ($3.00 each); 45 and 43 frames.

Description of FS. 157: First there is explanation and practice in telling time to the hours and half-hours; then a series of pictures following Roger through the day, with a clock in the corner of each picture

for the time to be read; and finally six frames with clock faces and directions telling the pupils what time to indicate by drawing the hands on the frames projected onto the blackboard.

Description of FS. 158: After a review of telling time to the hours and half-hours an explanation of the sixty minutes and how they are read is given. Then the concepts of AFTER and TO with respect to the nearest hour are illustrated. We then see Jean and Paul going through the day with pictures of their activities and clock faces to read. Finally, there are six frames to have their hands drawn on as they are projected.

Appraisal of FS. 157 and FS. 158: This is a good set of pictures for its purpose. It should certainly be supplemented by a clock face which can be manipulated. However, there has been a real attempt to get participation into the types of frames used. It should be a help to associate the various times told by the clock with the usual activities of a child's life, rather than try to teach time as an abstract activity on a clock face.

FS. 159—A Study of Measurement

Photo and Sound Productions, 116 Natoma Street, San Francisco 5, Calif.

Educational Collaborator, O. W. McGuire B&W ($4.00); 30 frames.

Description: Among the standards of measurement portrayed are: digit from the breadth of the little finger, the inch from the breadth of the thumb, the palm or hand from the four fingers, the span from the tip of the thumb to the tip of the outstretched finger, the cubit from the distance of the point of the elbow to the tip of the little finger, the pace or double step, the foot from the length of the human foot, the rod from the combined length of the left feet of sixteen men, the rod from the length of the ox goad, the acre from 40 ox goads by 4 ox goads, the inch from three grains of barley, and the yard from the distance around the human body.

Appraisal: The upper elementary, junior high school and high school students will be interested in this filmstrip. As a brief pictorial representation of the history and development of measurement, it has a number of excellent pictures. The filmstrip has a number of excellent frames showing important ideas such as the need to standardize the different standards of measure.

The filmstrip can be used as an aid to the introduction of measurement; as an illustration while teaching; and a review of the development of measurement. The amount of ground covered is just right; the material is accurate and authentic.

The filmstrip interested the students and they conducted some interesting comparisons of their own "standard of measurement" by comparing their feet, fingers, etc. (Reviewed by Perry Chapdelaine, Mason City, Iowa.)

FS. 160—The Transit; Part I—Description, Set-Up and Leveling

FS. 161—The Transit; Part II—Verniers

United World Films, Inc., 1445 Park Ave., New York 29, N. Y.

B & W ($.72 each less school discount)

Description of FS. 160: The various parts of a transit are described and illustrated and their functions explained. The use of a tripod in setting up is carefully outlined, and the correct procedure for leveling the transit concludes this introduction to the use of a transit. The illustrations and some of the warnings are in terms of the military use of this instrument.

Description of FS. 161: The filmstrip on verniers tells how to determine the least reading of a vernier, how to read the main scale, how to read the vernier scale and how to combine these for the complete reading. Then twelve problems are presented for the student to read. The frame following each problem repeats the problem with the answer.

Appraisal of FS. 160 and FS. 161: The

small cost of these filmstrips certainly compensates for the military atmosphere they create with the illustrations and directions. They are very well organized. The amount of detail is too much for most secondary school mathematics classes, but it need not be taught nor discussed; it certainly shows mathematics doing a practical job. Of the two filmstrips the second has more material which would lead to mathematical concepts such as scale reading, division of fractions, addition and subtraction of numbers, denominate numbers, and units of measurement. The first filmstrip would be meaningless without a transit to handle, the second would be better with the instrument than without it.

MATHEMATICS TESTS

Edited by JOHN H. HAYNES

Acton High School, West Acton, Massachusetts

REVIEWS

Davis Test of Functional Competence in Mathematics. David J. Davis; Evaluation and Adjustment Series, Walter N. Durost, editor; World Book Co.; $2.50 per pack of 25; $.35 per copy; 1951; end of the year achievement; Forms AM and BM; 40 minute periods; grades 9–12; "to measure functional competence in mathematics."

Description: This test of 80 items is split into two parts. Part I consists of 33 multiple-choice items which test the student's ability to solve verbal problems which occur in everyday life, and require a knowledge of topics such as insurance, installment buying, banking, profit-cost-selling price relationship, taxes, and others. Also included are questions which test ability to read tables and graphs of various types. The 47 multiple-choice items of Part II include work on formulas, the use of letters in simple algebra, areas, angles, approximate numbers, scale drawing, fractions and decimals, use of a ruler, and ability to draw a valid conclusion from a set of facts. The test booklets are well printed, easily read, and directions are clear.

Validity: A list of 16 basic objectives was drawn up from an examination of many writings on essential mathematics for all students. Test items were then written and administered, the results analyzed, difficulty data secured, and the final forms constructed. Statistical validation was also carried out.

Reliability: Split-half reliability coefficients ranging from .81 to .91 were found in 8 cases, 2 in each grade, from various schools. Further statistics show that for grade 9 there are two chances out of three an individual's score will not differ from his hypothetical "true" score by more than 5.9 points. This figure for grade 12 is 4.9.

Administration: The test is split so that it may be given in two class periods of 40 minutes working time each. Directions are clear and well written. Separate answer sheets may be used. A ruler is needed for Part II, and scratch paper should be supplied for each part.

Scoring: This test may be scored by machine or by hand. A punched key is supplied for easy hand scoring.

Interpretation: The raw scores on this test, as on others in this series, are converted to scaled scores which are associated with the distribution of scores on the Terman-McNemar IQ Test for the grade 10 pupils on which the test was standardized. Percentile scores for grades 9–12 are given. These norms are based on 1,464 grade 9, 1,179 grade 10, 1,136 grade 11, and 1,371 grade 12 students from 19 states. These percentiles are given for both the midyear and the end of the year. The test is best used to determine individual achievement of basic mathematics objectives. The test is not diagnostic nor does it test mastery of various mathematical skills.

Conclusion: This test is a good one to test thinking in mathematics. In this connection it might be used profitably in general non-college mathematics classes. A teacher in such a situation must, however, be sure the required algebraic, geometric, and trigonometric ideas included in the test have been taught. It should also be useful to measure progress of students from one year to the next if administered at the end of each year.

BOOK SECTION

Edited by JOSEPH STIPANOWICH
Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

High School

Teachers Manual with Answers for Mathematics, A Second Course, by Myron F. Rosskopf, Harold D. Aten and William D. Reeve. Paper, 64 pages, 1952. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y.

General Trade Mathematics (Second ed.), by Edwin P. Van Leuven, formerly of Bakersfield High School and Bakersfield College. Cloth, viii+553 pages, 1952. McGraw-Hill Book Co., 330 West 42nd Street, New York 36, N. Y. $3.80.

College

The Higher Arithmetic, An Introduction to the Theory of Numbers, by H. Davenport, University of London. Cloth, viii+172 pages, 1952. Longmans, Green and Co., 55 Fifth Ave., New York 3, N. Y. $2.25 (Text ed., $1.80).

Statistical Theory in Research, by R. L. Anderson, University of North Carolina; and T. A. Bancroft, Iowa State College. Cloth, xix+399 pages, 1952. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $7.00.

Methods of Statistical Analysis (Second ed.), by Cyril H. Goulden. Cloth, vii+467 pages, 1952. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. $7.50.

Miscellaneous

What Does Research Say About Arithmetic? by Vincent J. Glennon and C. W. Hunnicut, both of Syracuse University. Paper, iv+45 pages, 1952. Association for Supervision and Curriculum Development, 1201 Sixteenth St., N.W., Washington 6, D. C. $0.50.

Armed Services Examinations: Mathematics, by Elmer A. Hable. Paper, 39 pages, 1952. Elmer A. Hable, Pensacola Junior College, Pensacola, Florida.

Financing Adult Education in Selected Schools and Community Colleges (Office of Education, Federal Security Agency Bulletin 1952, No. 8), by Homer Kempfer and William R. Wood. Paper, iv+27 pages, 1952. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. $0.15.

The Teaching of General Biology in the Public High Schools of the United States (Office of Education, Federal Security Agency Bulletin 1952, No. 9), by W. Edgar Martin. Paper, x+46 pages, 1952. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. $0.20.

Accounting Problems with Answers (College Outline Series) by Joseph C. Schabacker, University of California at Los Angeles; and Paul K. Schroeder, Whittier College. Paper, xii+212 pages, 1952. Barnes and Noble, Inc., 105 Fifth Ave., New York 3, N. Y. $1.50.

REVIEWS

Algebra: Its Big Ideas and Basic Skills (Book II), Daymond J. Aiken and Kenneth B. Henderson. New York, McGraw-Hill Book Company, 1952. xii+397 pp. $2.72.

The authors introduce advanced algebra to the students by extending their knowledge of numbers to include irrational, imaginary, and complex numbers. The topic of special products, factoring, and fractions is considered next, with what might be called "advanced topics in factoring" omitted. The material in this chapter should present little more difficulty to the student than that presented in the first year of algebra.

The book has an excellent chapter on logarithms. A student could probably understand and learn to use logarithms without an instructor if he followed carefully the development in this chapter.

Explanations throughout the book are good and should be valuable not only to the student but to the teacher as well. They should be of great assistance to the beginning teacher and in many cases provide different ideas of approach for the experienced teacher.

The entire last half of the book uses the function concept and includes discussion of linear, quadratic, general polynomial, and trigonometric functions, and progressions.

Illustrations are used not only to clarify problems but to urge students to acquire good study habits. Important ideas, including ways to avoid certain errors, are stressed by statements in bold face type enclosed in boxes. The choice of type and arrangement of the material in the book make it very legible.—MARIE S. WILCOX, George Washington High School, Indianapolis, Indiana.

Guiding Children's Arithmetic Experiences, J. Allen Hickerson. New York, Prentice Hall, Inc., 1952. xii+322 pp., $5.00.

This book, organized into three synchronized

parts, emphasizes successively (1) the nature of arithmetic and the ingredients of quantitative relationships necessary to (2) help children understand, use and compute with whole numbers and (3) to help children understand, use and compute with fractions and decimals.

Arithmetic as arithmetic is not emphasized but rather the "experience language" approach is used because of the author's belief that it is the most helpful in working with children to contribute to their mental growth and development.

Instead of discussing the topics and materials on a horizontal plan—grade by grade, the treatment follows a vertical plan, especially in dealing with the four fundamental operations. Such a plan involves beginning with the easiest elements of a topic, addition for example, in the lower grades and carrying this through varying levels of difficulty to the higher grades. This reviewer has used this type of organization in lectures to student teachers, for a period of several years, and finds it desirable in such situations. There should be some question about the value of the plan for children since their backgrounds are not comparable. The author does not recommend the vertical development for children but makes it for such readers as teachers, adults and student teachers so that these may get an over-all view of the breadth and integration of the topics that go to make up the usual field of arithmetic.

The mathematical thinking involves (1) the knowledge and use of mathematical symbols to represent quantity in concrete situations, (2) computation, and (3) an attempt at understanding the science and theory of mathematics. Additional help is found in the three Appendices. Appendix A—In-and-Out-of-School Uses of Arithmetic—presents, by grades, many worthwhile helps. Appendix B—Age-Placement of Computational Steps—presents data with which all may not agree but the inexperienced will find helpful nevertheless. Appendix C—Suggested Readings—includes annotations on recently published worthwhile books.

This book should prove helpful for those who work in the field of arithmetic.—Joseph J. Urbancek, Chicago Teachers College, Chicago, Illinois.

Calculus, John F. Randolph. New York, The Macmillan Company, 1952. x+483 pp., $5.00.

A function is defined by the author as "a collection of ordered pairs such that no two distinct ordered pairs of the collection have the same first element." The author continues to present the usual course material in a somewhat unusual but very effective manner. The concept of a limit is discussed in connection with special functions which amply illustrate the principles but the actual proofs for the limit theorems are included in the appendix. The limit of the sine function is introduced early to aid the teacher of physics whose work in harmonic motion may then draw upon this concept.

Definite integrals with their applications are dealt with extensively before the indefinite integral is fully discussed, although the author introduces the idea of an anti-derivative along with the concept of the derivative.

A chapter on solid analytic geometry preceding the study of partial derivatives and multiple integration is a welcome addition to the textbook, but the usual chapters on differential equations and hyperbolic functions are not included. Methods of finding approximations to integrals are treated extensively. Sequences and series with some applications complete the topics taken up.

While the text probably contains more material and certainly more theory than is found in the usual textbook in calculus and certainly more material than could be covered in the usual course, the clearness and completeness of each presentation is excellent. The book certainly merits consideration by the teacher who wishes a textbook which goes beyond the usual in attempting to give adequate explanations to the concepts developed in the beginning course in calculus. It contains excellent reference material for the student and the teacher as well.—Herbert Hannon, Western Michigan College of Education, Kalamazoo, Michigan.

Matter and Motion, James Clerk Maxwell. New York, Dover Publications Inc. 178 pp., $1.25 (paper) or $2.50 (cloth).

This is a reprint of the 1887 edition of Maxwell's splendid classic in elementary mechanics. It also contains a chapter on The Lagrange Equations reprinted from the author's *Treatise on Electricity and Magnetism* (1873), as well as two appendices by Sir Josef Larmor, one on the Principle of Least Action. Outside of these, the book is quite elementary; indeed, it does not use any calculus. The discussion of fundamental principles are extremely lucid and reveal Maxwell's great art as a teacher. The book would be excellent reading for anyone starting to learn physics and certainly for teachers. The greater part of it is devoted to a discussion of the fundamental concepts of force, mass, energy, etc., and works up to Kepler's Laws and Newton's Law of Gravitation. Special note should be made of the fact that the book is available in a paper-bound edition. The publishers are to be highly commended for this.—W. E. Jenner, Northwestern University, Evanston, Illinois.

The Philosophy of Mathematics, Edward A. Maziarz. New York, Philosophical Library, 1950. viii+286 pp., $4.00.

Here is a scholarly and carefully documented discussion for those who wish to attain perspective in mathematical thinking. True to one of the purposes of philosophy, it enables the reader, figuratively speaking, to climb a hill and from this vantage point survey mathematics as a whole and the interrelationship of its various parts. Those who are interested in the subject find this necessary to do from time to time be-

cause of the proliferation resulting from the research in the subject. One can easily lose sight of the forest because of the abundance of the interesting trees.

The book begins by briefly setting the problem of the philosophy of mathematics in a wider context of relevant speculative thinking. The rest of the book is organized into two parts. The first part, consisting of six chapters, is a history of the inquiry, "What is mathematics?" In his treatment of this historical development, the author draws on the deliberations of both philosophers and mathematicians, and sweeps the interval from the Pythagoreans to contemporary philosophers of mathematics. The second part of the book, consisting of three chapters, considers the epistemological and metaphysical questions emerging from a philosophical analysis of the nature of mathematics. The discussion of abstraction in mathematics is complete and penetrating. It is in this part that the author presents his thesis that, "The nature of mathematical science is unified and specified by its ultimate subject matter, quantity, or the order of the parts of quantified substance."

Prerequisite knowledge for reading the book appears to be more in terms of philosophical understanding than mathematical understanding. The style is not "popularized"; at times it is difficult reading. But profound ideas cannot be popularized and still be cogent and rigorous.—Kenneth B. Henderson, University of Illinois, Urbana, Illinois.

General Education in Science, I. Bernard Cohen and Fletcher G. Watson (With a foreword by James B. Conant). Cambridge, Harvard University Press, 1952. xviii+217 pp., $4.00.

Sixteen distinguished scientists and science teachers, most of them members of the Harvard faculty, have pooled their ideas on the teaching of science within the framework of General Education. The outcome is most informative and stimulating. It reveals a deep concern among science teachers about the shortcomings of existing science courses in colleges and secondary schools, and it suggests a variety of remedies. Some of these, such as the infusion of philosophy, history, and sociology into science courses, would seem applicable to courses in mathematics. The book should, therefore, be useful to the mathematics teacher.

This reviewer, however, was disappointed to find no more than the most fugitive references to mathematics, and he wonders how the evolution of science can be traced for students without an adequate treatment of the contribution of mathematics.—Paul R. Neureiter, State University Teachers College, Geneseo, N. Y.

The Nature of Number (An Approach to Basic Ideas of Modern Mathematics), Roy Dubisch. New York, The Roland Press Company, 1952. xii+159 pp., $4.00.

This book should be of interest to those teaching (or preparing to teach) mathematics, especially at the college level. The objectives are as follows: (1) "To portray the development of a single line of mathematical thought from its most primitive beginnings to contemporary times, and (2) to make the abstractions of advanced mathematics easier to grasp by pointing out that the concreteness of elementary mathematics is largely illusory and that, actually, all basic mathematics is abstract."

To get this over-all view the reader is led down a narrow path from the very primitive mathematical ideas involved in counting and arithmetic to a discussion of the structure theory of hypercomplex systems (or, the so-called linear algebra). *Cayley* numbers and *Jordan* algebra are discussed as examples of the relatively new nonassociative algebras.

The style of the book is quite informal and is loaded with historical references. Included at the end of each chapter are problems for the reader to solve to help in his understanding of the new concepts.

The content by chapters: "Counting from One to a Googol," "Writing Numbers from One to a Googol," "God Made the Integers," "Creation Completed," "Zero Again—and Less," "Fractions are Easy Now," "The Unspeakable," "The Great *i* Solves All," "A Shot in the Arm for Complacent Algebraists of the Nineteenth Century," "How Lasting Was the Shot." "Last-minute Edition of the *Algebra Gazette*," and "The Author's Lament."

The text also contains an appendix, suggestions for further reading, and answers to the problems proposed.—Joseph Stipanowich, Western Illinois State College, Macomb, Illinois.

Calculus, Atherton Hall Sprague. New York, Ronald Press, 1952. xi+576 pp., $6.50.

This text follows the classical order of material in that the first portion of the book is devoted to the differential calculus of one variable, followed by integral calculus, infinite series, and the calculus of functions of two and three variables. There is a slight variation from this pattern in that the idea of antiderivative is introduced briefly in the second chapter along with the discussion of derivatives. A chapter on solid analytics and determinants is included.

Careful attention is given to the definition and illustration of basic concepts. Problem solving techniques are well explained by many detailed examples. The problem lists appear to be adequate. The usual geometric and physical applications of differential and integral calculus are included. The vector notation is employed in the study of curvilinear motion.

Answers to most of the exercises are found in the back of the book. The printing is good and the format attractive.

The book should be suitable for any beginning class in calculus, although it is perhaps more definitely pointed towards students of pure mathematics rather than engineers.—Cleon C. Richtmeyer, Central Michigan College of Education, Mt. Pleasant, Michigan.

Program
The National Council of Teachers of Mathematics
Thirty-First Annual Meeting

Ambassador Hotel, Atlantic City, New Jersey
April 8, 9, 10, 11, 1953

Host Organization: Association of Mathematics Teachers of New Jersey
Convention Theme: Increasing Student Participation in Mathematics Classes

REGISTRATION

Foyer on the Lounge Floor

Wednesday, April 8, 7:00–9:00 P.M.

Thursday and Friday, April 9 and 10, 8:00 A.M.–9:00 P.M.

Saturday, April 11, 8:00 A.M.–12:00 NOON

SCHOOL AND COMMERCIAL EXHIBITS

Lounge Floor Promenade and the Rotunda

Thursday, April 9, 9:00 A.M. to Saturday, April 11, 12:00 NOON

SECTIONS OF GENERAL INTEREST

WEDNESDAY, APRIL 8

9:00 A.M.–3:30 P.M. Visiting Atlantic City Schools

(See announcement at the end of the program.)

THURSDAY, APRIL 9

9:00 A.M.–12:00 NOON. Visiting Atlantic City Schools

(See announcement at the end of the program.)

10:30–11:50 A.M.—22-Club

Showing of Mathematics Films

1:30–3:30 P.M.—Room 125

This section is sponsored by the Committee on Research of the N.C.T.M.

Semantic Aspects of Arithmetic, IRENE HARRISON, New York University, New York, N. Y.

The Beginning Instructor and the Teaching Profession, W. C. STONE, Union College, Schenectady, N. Y.

An Enriched Mathematics Program for Future Scientists and Engineers, ANDREW V. KOZAK, Concord College, Athens, West Virginia

1:30–3:30 P.M.—Renaissance Room

The Contributions the U. S. Office of Education May Make to Mathematics Education, KENNETH E. BROWN, Specialist for Mathematics, Office of Education, Washington, D. C.

Mathematical Concepts and Skills Which Contribute to Science Teaching, MORRIS MEISTER, Principal, Bronx High School of Science, New York, New York

Teacher Encouragement, W. W. RANKIN, Duke University, Durham, North Carolina

4:00 P.M.—22-Club

Showing of Mathematics Films

7:30–9:00 P.M.—Renaissance Room

Welcome to New Jersey, CARL N. SHUSTER, State Teachers College, Trenton, New Jersey

The Twenty-First Yearbook, HOWARD F. FEHR, Editor, Teachers College, Columbia University, New York, New York

The Future of Mathematics Education, in the Secondary School, WILLIAM D. REEVE, New York, New York

9:00–10:00 P.M.—Lounge

Reception, Association of Teachers of Mathematics of New Jersey

FRIDAY, APRIL 10

8:45–10:00 A.M.—Renaissance Room

Mathematics and the Smithsonian Institution, LEONARD CARMICHAEL, Head of the Smithsonian Institution, Washington, D. C.

10:30 A.M.–12:00 NOON—Room 122

Infinity, In Elementary and Secondary

Mathematics, HENRY W. SYER, Boston University, Boston, Massachusetts

Some Concepts in Abstract Algebra Which Tend to Illuminate Elementary Mathematics, E. A. CAMERON, The University of North Carolina, Chapel Hill, North Carolina

4:40–5:40 P.M.—22-Club

Showing of Mathematics Films

7:00 P.M.—Renaissance and Venetian Rooms

New Jersey Shore Dinner

Professional Opportunities in Mathematics, MINA REES, Director, Mathematical Sciences Division, Office of Naval Research, Washington, D. C.

SATURDAY, APRIL 11

9:30–10:30 A.M.—Renaissance Room

Reason and Rule in Arithmetic and Algebra, RALPH BEATLEY, Harvard University, Cambridge, Massachusetts

10:45–12:00 NOON—Room 122

Statistical Quality Control (A report of the United Nations Technical Assistance Mission to India), PAUL C. CLIFFORD, State Teachers College, Montclair, N. J.; ELLIS R. OTT, Rutgers University, New Brunswick, N. J., MASON WESCOTT, Rutgers University, New Brunswick, N. J.

10:45 A.M.–12:00 NOON—Room 117

Problems of the Mathematics Supervisor

Chairman and Discussion Leader: ROLLAND R. SMITH, Springfield, Mass.

Speakers: HERSCHEL GRIME, Directing Supervisor of Mathematics, Cleveland, Ohio; BERNADINE M. JONES, Curriculum Consultant in Mathematics, Montgomery County, Maryland; MILDRED KEIFFER, Mathematics Supervisor, Cincinnati, Ohio; ELIZABETH ROUDEBUSH, Mathematics Supervisor, Seattle, Washington.

12:30 P.M.—Renaissance and Venetian Rooms

Luncheon

Master of Ceremonies: HOWARD F. FEHR, Teachers College, Columbia University, New York, New York

Speaker: HERMAN VON BARAVALLE, Adelphi College, Garden City, New York

2:30–4:00 P.M.—Room 125

Affiliated Groups Section

Panel Discussion: *Meeting the Needs of the Gifted Student*

Chairman: DONOVAN A. JOHNSON, University of Minnesota, Minneapolis, Minnesota

Participants: KENNETH E. BROWN, Mathematics Specialist, U. S. Office of Education, Washington, D. C.; MYRL H. AHRENDT, Executive Secretary, NCTM, Washington, D. C.; JOHN SCHACHT, Bexley High School, Columbus, Ohio; MAMIE AUERBACH, John Marshall High School, Richmond, Virginia; C. LOUIS THIELE, Divisional Director, Exact Sciences, Detroit, Michigan

ELEMENTARY SCHOOL SECTIONS

THURSDAY, APRIL 9

1:30–3:30 P.M.—22-Club

What Constitutes Meaningful Instruction in Arithmetic in the Primary Grades? JOYCE BENBROOK, University of Houston, Houston, Texas

The Appraisal and Use of Devices in Arithmetic, BEN A. SUELTZ, State Teachers College, Cortland, New York

Helping Children Build a Positive Attitude Toward Arithmetic, CLARENCE ETHEL HARDGROVE, Northern Illinois State Teachers College, DeKalb, Illinois

1:30–3:30 P.M.—Rooms 110 and 111

Elementary School Laboratory

Director: CATHERINE M. WILLIAMS, Ohio State University, Columbus, Ohio

FRIDAY, APRIL 10

10:30 A.M.–12:00 NOON—22-Club

Elementary School Demonstration Lesson

Teacher: IDA MAE HEARD, Southwest-

ern Louisiana Institute, Lafayette, Louisiana

Topic: *Your State and Mine* (6th Grade)

10:30 A.M.-12:00 NOON—Surf Room

Sequential Development of the Ability to Use Round Numbers and Estimate Answers in Grades One Through Eight, CHARLOTTE JUNGE, Wayne University, Detroit, Michigan

Enrichment Activities for Superior Pupils in Grades 5 and 6, MILDRED COLE, C. M. Bardwell School, Aurora, Illinois

What Concepts Should Be Emphasized in the Algebra and Geometry of Grades Seven and Eight? HELEN A. SCHNEIDER, Oak School, La Grange, Illinois

1:30-2:45 P.M.—Embassy Room

Elementary Continuity Group—Part I

Topic: *Building Mathematical Concepts in Grades One to Six Through Guided Experiences*

Speaker-Analyst: EDWINA DEANS, Arlington County Public Schools, Arlington, Virginia

1:30-2:45 P.M.—Rooms 121 and 122

Topic: *Use of Drill in Learning Arithmetic*

Discussion Leader: BEN A. SUELTZ, State Teachers College, Cortland, New York

3:00-4:30 P.M.—Embassy Room

Elementary Continuity Group—Part IIa

Discussion Leader: OLIVE WEAR, Elementary Supervisor, Fort Wayne, Indiana

3:00-4:30 P.M.—Room 110

Elementary Continuity Group—Part IIb

Discussion Leader: JEAN HAMILTON, Wayne University, Detroit, Michigan

3:00-4:30 P.M.—Room 105

Flexibility in the Arithmetic Program to Provide for Maximum Pupil Growth, MAUDE COBURN, Oakland Public Schools, Oakland, California

Critical Thinking as a Goal in the Teaching of Arithmetic, LOWRY HARDING, Ohio State University, Columbus, Ohio

SATURDAY, APRIL 11

10:45 A.M.-12:00 NOON—Embassy Room

Elementary Continuity Group—Part III

Topic: *Building Mathematical Concepts in Grades One to Six Through Guided Experiences*

Speaker-Analyst: EDWINA DEANS

10:45 A.M.-12:15 P.M.—Room 125

Essential Understandings for Teachers of Arithmetic in Grades 1-4, MARGUERITE BRYDEGAARD, San Diego State College, San Diego, California

Essential Understandings for Teachers of Arithmetic in Grades 5 and 6, PETER SPENCER, Claremont College, Claremont, California

2:30-4:00 P.M.—Embassy Room

The Nature of Children's Thinking in Quantitative Situations, GLENADINE GIBB, The University of Wisconsin, Madison, Wisconsin

The Role of Pupil Discovery in the Development of Problem Solving Ability, E. W. HAMILTON, Iowa State Teachers College, Cedar Falls, Iowa

Important Concepts for the Upper Grades, LENORE JOHN, The University of Chicago, Chicago, Illinois

JUNIOR HIGH SCHOOL SECTIONS

THURSDAY, APRIL 9

1:30-3:30 P.M.—Rooms 117 and 118

Junior High School Laboratory

Director: ELEANOR E. TAYLOR, Central Junior High School, Quincy, Massachusetts

Assistant: GERTRUDE C. HAZZARD, Guilford High School, Guilford, Connecticut

1:30-3:30 P.M.—Surf Room

Children Should be Seen and Heard, MARY A. POTTER, Supervisor, Public Schools, Racine, Wisconsin

Solving Algebra Problems—A Step Toward Orderly Thinking, EDITH WOOLSEY, Sanford Junior High School, Mineapolis, Minnesota

Fitting the Curriculum to the Child, DOROTHY SWARD, Roosevelt Junior High School, Beloit, Wisconsin

FRIDAY, APRIL 10

10:30 A.M.–12 NOON—Venetian Room
Junior High School Demonstration Lesson
Topic: *The Micrometer*
Teacher: MARY CURTIS, Hampstead Hill Junior High School, Baltimore, Maryland

10:30 A.M.–11:55 A.M.—Rooms 104 and 105
Techniques of Developing Pupil Responsibility in Mathematics, ALICE M. HACH, Junior High School, Ann Arbor, Michigan
Some Ways of Promoting Learning, ROSE KLEIN, Somers Junior High School, Brooklyn, New York

1:30–2:45 P.M.—Surf Room
Junior High School Continuity Group—Part I
Topic: *The Student, Content and Goals of Mathematical Education in the Junior High School—Some Unanswered Problems for Discussion*
Speaker-Analysts: HOWARD F. FEHR, Teachers College, Columbia University, New York, New York, and VERYL SCHULT, Public Schools, Washington, D. C.

1:30–2:45 P.M.—Room 105
Problems of Articulating the Elementary Arithmetic Program With That of the Secondary Mathematics Program
Discussion Leader: EDITH WALKER, Director of Intermediate Education, Baltimore, Maryland

3:00–4:30 P.M.—Room 118
Developing Confidence in Mathematics Through Experimentation, PHILIP PEAK, Indiana University, Bloomington, Indiana
Specific and General Motivating Devices Used in Algebra, MILTON BECKMANN, University of Nebraska, Lincoln, Nebraska

3:00–4:30 P.M.—Surf Room
Junior High School Continuity Group—Part IIa
Topic: *Building Meaningful Concepts in Junior High School Mathematics*
Leader: NANETTE R. BLACKISTON, Supervisor, Public Schools, Baltimore, Maryland
Secretary: ANNIE JOHN WILLIAMS, Carr Junior High School, Durham, North Carolina
Evaluator: IDA MAY BERNHARD, Texas Education Agency, Austin, Texas

3:00–4:30 P.M.—Room 122
Junior High School Continuity Group—Part IIb
Topic: *Basic Learnings in the Junior High School*
Leader: ETHEL H. GRUBBS, Head of Mathematics Department, Division 2, Washington, D. C.
Secretary: ROSE KLEIN, Somers Junior High School, Brooklyn, New York
Evaluator: MARGARET DUNN, Bloomfield, Junior High School, Bloomfield, New Jersey

3:00–4:30 P.M.—Room 106
Junior High School Continuity Group—Part IIc
Topic: *Providing for Individual Differences*
Leader: ROLLAND R. SMITH, Coordinator of Mathematics, Public Schools, Springfield, Massachusetts
Secretary: HELEN ROBERTSON, Eastern High School, Washington, D. C.
Evaluator: RUTH GREEN, P. S. Du Pont High School, Wilmington, Delaware

SATURDAY, APRIL 11

10:45–11:50 A.M.—Surf Room
Junior High School Continuity Group—Part III
Topic: *The Student, Content and Goals of Mathematical Education in the Junior High School—A Possible Perspective*
Speaker: VERYL SCHULT, and HOWARD F. FEHR

10:45–11:55 A.M.—Room 116
Planning Mathematics Assembly Programs
Discussion Leader: JULIA E. DIGGINS, Paul Junior High School, Washington, D. C.

2:30–4:00 P.M.—Rooms 121 and 122

Cooperative Learning Activities in Junior High School Mathematics, JOHN J. KINSELLA, New York University, New York, New York

Improved Experiences in Problem Solving Result From More Student Participation, ROBERT E. PINGRY, University of Illinois, Urbana, Illinois

SENIOR HIGH SCHOOL SECTIONS

THURSDAY, APRIL 9

1:30–3:30 P.M.—Rooms 121 and 122

Senior High School Laboratory

Director: AMELIA RICHARDSON, McKeesport High School, McKeesport, Pennsylvania

1:30–3:30 P.M.—Venetian Room

The Role of Space Perception in the Teaching of Geometry, ERNEST R. RANUCCI, Weequahic High School, Newark, New Jersey

Enrichment Topics for Senior High School Mathematics Classes, MARGARET JOSEPH, Shorewood High School, Milwaukee, Wisconsin

The Sum and Difference Formulas in Trigonometry, HARRY RUDERMAN, Manual High School of Aviation Trades, New York, New York

1:30–3:00 P.M.

Tenth Grade Mathematics for Those Who Are Not Taking Plane Geometry or Second Year Algebra

Discussion Leader: HOWARD L. GALLANT, Hillsborough High School, Tampa, Florida

FRIDAY, APRIL 10

10:30–12:00 NOON—Renaissance Room

Senior High School Demonstration Lesson

Teacher: JULIUS H. HLAVATY, Bronx High School of Science, New York, New York

10:30–12:00 NOON—Room 108

Secondary School Laboratory

Director: GEORGE W. KAYS, United States Military Academy, West Point, New York

10:30–12:00 NOON—Room 125

Panel Discussion: *Integration of Subject Matter in the Conventional Course in High School Mathematics*

Participants: JACKSON B. ADKINS, Phillips Exeter Academy, Exeter, New Hampshire; MAX BEBERMAN, University High School, Urbana, Illinois; WILLIAM A. GAGER, University of Florida, Gainesville, Florida; BARNETT RICH, Richmond Hill High SCHOOL, Richmond Hill, New York; F. LYNWOOD WREN, George Peabody College for Teachers, Nashville, Tennessee

1:30–2:45 P.M.—22-Club

Senior High School Continuity Group—Part I

Topic: *Student Participation in the Mathematics Class*

Speaker-Analyst: PHILLIP S. JONES, University of Michigan, Ann Arbor, Michigan

1:30–2:45 P.M.—Room 118

Can Students Learn to Create Their Own Originals in Geometry?

Discussion Leader: JOHN SCHACHT, Bexley High School, Columbus, Ohio

1:30–2:45 P.M.—Room 110

Procedures in Classroom Management

Discussion Leader: CATHERINE A. V. LYONS, Perry High School, Pittsburgh, Pennsylvania

1:30–2:45 P.M. Room 108

We Live in a World of Mathematics

Discussion Leader: JEANNETTE GARRETT, Phillips High School, Birmingham, Alabama

3:00–4:30 P.M.—22-Club

Senior High School Continuity Group—Part IIa

Topic: *Student Participation in the Algebra Class*

Leader: J. HOUSTON BANKS, George Peabody College for Teachers, Nashville, Tennessee

3:00–4:30 P.M.—Room 109

Senior High School Continuity Group—Part IIb

Topic: *Student Participation in the Geometry Class*
Leader: Nathan Lazar, Ohio State University, Columbus, Ohio
3:00–4:30 p.m.—Room 125
The Use of the Historic Approach to Secondary Mathematics, Herta Taussig Freitag, Hollins College, Hollins College, Virginia
Fixed Point Theorems, Robert C. James, Haverford College, Haverford, Pennsylvania

Saturday, April 11

10:45–11:55 a.m.—22-Club
Senior High School Continuity Group—Part IIIa
Topic: Continuation of Part IIa
Leader: J. Houston Banks
10:45–11:55 a.m.—Room 109
Senior High School Continuity Group—Part IIIb
Topic: Continuation of Part IIb
Leader: Nathan Lazar
10:45–11:55 a.m.—Rooms 104 and 105
Panel Discussion: *A Re-Examination of General Mathematics*
Chairman: Francis G. Lankford, Jr., University of Virginia, Charlottesville, Virginia
Participants: W. A. Gager, University of Florida, Gainesville, Florida; George E. Hawkins, Lyons Township High School and Junior College, LaGrange, Illinois; Mary A. Potter, Mathematics Consultant, Racine, Wisconsin; C. Louis Thiele, Divisional Director, Exact Sciences, Detroit, Michigan
10:45–11:55 a.m.—Room 120
Special Emphases in Algebra
Discussion Leader: Isabel H. Kinnett, Lanier High School for Boys, Macon, Georgia
2:30–4:00 p.m.—22-Club
Visualization in Algebra, Maurice L. Hartung, University of Chicago, Chicago, Illinois
Dear Teacher (Open letters to teachers of mathematics in small high schools)
Participants: Mabel Love Baker, Penn Township Schools, Pittsburgh, Pennsylvania; John A. Brown, Wisconsin High School, Madison, Wisconsin; Gertrude Hendrix, Eastern Illinois State College, Charleston, Illinois; Houston T. Karnes, Louisiana State University, Baton Rouge, Louisiana; M. Albert Linton, William Penn Charter School, Philadelphia, Pennsylvania; H. Vernon Price, State University of Iowa, Iowa City, Iowa

COLLEGE AND TEACHER EDUCATION SECTIONS

Thursday, April 9

1:30–2:30 p.m.—Room 105
Student Teaching in Mathematics
Discussion Leader: Gilbert Ulmer, Kansas University, Lawrence, Kansas
2:45–3:45 p.m.—Room 105
College Algebra, What Is It?
Discussion Leader: Nellie Pyle Miser, Lebanon, Illinois

Friday, April 10

10:30 a.m.–12:00 Noon—Rooms 117 and 118
Training Teachers via the College Mathematics Laboratory, Daniel B. Lloyd, Wilson Teachers College, Washington, D. C.
Training the Basic Mathematics Teacher, Lee E. Boyer, State Teachers College, Millersville, Pennsylvania
Importance of Calculus in Teacher Education, Houston T. Karnes, Louisiana State University, Baton Rouge, Louisiana
1:30–2:45 p.m.—Venetian Room
College Continuity Group—Part I
Topic: *Teaching and Learning at the College Level*
Speaker-Analyst: Henry W. Syer, Boston University, Boston, Massachusetts
1:30–2:45 p.m.—Room 125
Theme: Student Participation in Courses for Teacher Training in Mathematics

Professionalized Subject Matter for Junior High School Mathematics Teachers, MYRON F. ROSSKOPF, Teachers College, Columbia University, New York, New York

A Device to Help Student Teachers of Mathematics Evaluate Their Own Teaching, ROBERT E. PINGRY, University of Illinois, Urbana, Illinois

Are Student Projects Practical in College Mathematics? H. C. TRIMBLE, Iowa State Teachers College, Cedar Falls, Iowa

3:00–4:30 P.M.—Rooms 102 and 103

Theme: Basic Elementary College Mathematics for Prospective Teachers of College Mathematics

Appraisal of Certain Basic Ideas in Elementary College Mathematics, WILSON L. MISER, McKendree College, Lebanon, Illinois

The Mathematical Preparation of College Teachers, BRUCE E. MESERVE, University of Illinois, Urbana, Illinois

The Implementation of Some Basic Ideas in Elementary College Mathematics, JOHN G. BOWKER, Middlebury College, Middlebury, Vermont

3:00–4:30 P.M.—Venetian Room

College Continuity Group—Part II

Teaching and Learning at the College Level

Speaker-Analyst: HENRY W. SYER

Discussion: This is a single group but will break up into smaller groups after the discussion gets under way.

Discussion Leaders: WILLIAM R. RANSOM, Tufts College, Medford, Massachusetts and RALPH BEATLEY, Harvard University, Cambridge, Massachusetts

SATURDAY, APRIL 11

10:45 A.M.–12:00 NOON—Room 106

College Continuity Group—Part III

Topic: *Teaching and Learning at the College Level*

Speaker-Analyst: HENRY W. SYER

Report of the discussion groups and final analysis

10:45 A.M.–12:15 P.M.—Room 102

Teacher Education from the Viewpoint of the High School Teacher, MARTHA HILDEBRANDT, Proviso Township High School, Maywood, Illinois

Providing for Individual Differences in Elementary College Mathematics, E. A. CAMERON, University of North Carolina, Chapel Hill, North Carolina

Selection of Courses for the Prospective Mathematics Teacher, JAMES J. BARRON, Marshall College, Huntington, West Virginia

2:30–4:00 P.M.—Surf Room

Panel Discussion: *The Need for Better Articulation Between Secondary Schools and Colleges to Provide Adequate Mathematical Training for Future Engineers, Mathematicians and Scientists*

Leader: C. V. NEWSOM, University of the State of New York, Albany, New York

Participants: E. B. ALLEN, Rensselaer Polytechnic Institute, Troy, New York; R. S. BURINGTON, Bureau of Ordinance, Navy Department, Washington, D. C.; GEORGE A. RIETZ, Educational Services, General Electric Company, Schenectady, New York; S. S. STEINBERG, University of Maryland, College Park, Maryland

2:30–4:00 P.M.—Room 110

The Master's Degree Program for Prospective Mathematics Teachers

Discussion Leaders: H. G. AYRE, Western Illinois State College, Macomb, Illinois; and CLEON C. RICHTMEYER, Central Michigan College of Education, Mt. Pleasant, Michigan

NATIONAL COUNCIL BUSINESS

WEDNESDAY, APRIL 8

9:00 A.M.–10:00 P.M.—Room 116

Meeting of the Board of Directors

THURSDAY, APRIL 9

8:00–11:00 A.M.—Embassy Room and Room 125

Breakfast for Delegates and Delegate Assembly

Presiding: MARY C. ROGERS, Roosevelt Junior High School, Westfield, New Jersey

11:00–11:55 A.M.—Room 125
Meeting of State Representatives
Presiding: MYRL H. AHRENDT, Executive Secretary, Washington, D. C.

FRIDAY, APRIL 10

8:00–10:00 A.M.—Room 125
Second Session of the Delegate Assembly
Presiding: MARY C. ROGERS

SATURDAY, APRIL 11

8:00–9:15 A.M.—22-Club
Annual Business Meeting

ANNOUNCEMENTS

Registration

The registration fee is fifty cents for members of the National Council of Teachers of Mathematics, members of the Mathematical Association of America, and for teachers in elementary schools. The fee for non-members and visitors is $1.50. Undergradute students sponsored by a faculty member, relatives of members, invited speakers who are not members, members of the press, and commercial exhibitors are not charged the registration fee but should register. You are urged to register in advance, and requested to check in at the Registration Desk upon your arrival at the meeting. Use the Advance Registration and Reservation Form to be found on page 137. Registrations and reservations received before March 25 will be acknowledged by return mail.

Room Reservations at The Ambassador

Convention rates are as follows:

European Plan—Daily Rates—Rooms with Private Baths

Single, 1 person $6, $8, $10, $12, $14

Double, 2 persons $8, $10, $12, $14, $16, $18

Parlor—1 Double with Bath $20, $28, $36

Parlor—2 Doubles, 2 Baths, $32, $42, $54

Additional Charge for third occupant of room—$3.00 daily

Applications should be sent directly to The Ambassador, Atlantic City, New Jersey.

Transportation

Atlantic City is accessible by train, plane, bus, or private car. Transportation from the Atlantic City Airport to Atlantic City is readily available. There are no direct transcontinental airplane connections with Atlantic City. Transfers must be made at the Newark, New Jersey, at the New York City, the Philadelphia or the Washington airports to the All-American Airlines which service Atlantic City.

There are two boulevards leading directly into Atlantic City. One, the Albany Avenue Boulevard, connects with the New Jersey Black Horse Pike from Camden, New Jersey and Philadelphia, Pennsylvania; the other, the Absecon Boulevard, connects with both the White Horse Pike (Camden, New Jersey and Philadelphia, Pennsylvania) and the New York Road. If one is travelling via the Black Horse Pike, the Albany Avenue Boulevard is nearest to the Ambassador Hotel. However, both boulevards are not too far from the hotel. There is a small parking lot adjacent to the hotel and there are nearby private parking concessions for which a fee is charged. The fees are moderate in the Spring.

Atlantic City is on a branch of the Pennsylvania Railroad. There are some train connections direct from New York City or from Southern areas. However, many of the train connections require changing trains in *North Philadelphia*.

Information

The Information Committee will furnish you with information on rooms, meals, parking, amusements, sight seeing trips by bus or boat, stores, schools, colleges, travel facilities, etc. A daily bulletin of special activities will be issued. When at the Convention, ask for help at the Information Desk.

Banquet and Luncheon Reservations

A New Jersey Shore Dinner will replace the usual Banquet on Friday night. The Convention Luncheon will be held on Saturday as is the usual custom. Reservations for the Shore Dinner and for the Luncheon should be made in advance. Requests should be accompanied with check or money order. All orders received before March 25 will be acknowledged by return mail. New Jersey Shore Dinner $4.60; Luncheon $3.20 tax and tips included. Use the Advance Registration and Reservation Form.

Visiting Atlantic City Schools

You are cordially invited to observe work in the Atlantic City schools on Wednesday, April 8 or on Thursday morning, April 9. There will be an opportunity to observe classes in mathematics in the elementary, junior and senior high schools. A detailed schedule will be available at the Information Desk, but requests to observe should be mailed in advance to Miss Eleanor Helfrich, Administration Building, 1809 Pacific Avenue, Atlantic City, New Jersey.

Continuity Groups

Four Continuity Groups, each meeting in three sessions planned as a continuous sequence, have been scheduled. The four Groups represent the elementary, junior high school, senior high school, and college levels. The three sessions for each of the Groups are to be held 1:30–2:45 P.M. Friday, 3:00–4:30 P.M. Friday, and 10:30–12 noon Saturday. In general the Continuity Group sessions include an address at the first session, followed by smaller discussion groups and a summary in the other two sessions. Those who desire to participate in one of the Continuity Groups are advised to register in advance for the Group of their choice.

Mathematics Laboratories

Four mathematics laboratories are planned. The plan of operation will include about a thirty-minute talk and demonstration on the construction and use of models in the classroom. Each member of a laboratory group will be given an opportunity to make several models, for which a minimum cost charge for materials used will be made. Refer to the program for further information. No individual will be permitted to participate in more than one laboratory session. Since the number who can attend any one of these sessions is very limited, advance registration is strongly advised.

Supplies and Equipment

Speakers and other participants on the program who need blackboards, projection equipment or other materials should communicate with Mrs. Wilma Nelson, Junior High School, Atlantic City, New Jersey before March 25.

Commercial Exhibits

Textbooks and commercial teaching aids will be on exhibit on the Lounge Floor Promenade from Thursday morning at 9 o'clock to noon on Saturday. Inquiries for exhibit space should be addressed to Max A. Sobel, 15–43 George Street, Fairlawn, New Jersey.

School Exhibits

There will be an exhibit of mathematics models, instruments, teaching aids, and other classroom materials on the Lounge Floor Promenade and in the Rotunda of The Ambassador from Thursday morning at 9 o'clock to Saturday noon. Teachers who wish to exhibit their materials are requested to communicate with Ernest R. Ranucci, Weequahic High School, Newark, New Jersey.

Meals for Special Groups

If any group wishes to arrange for a dinner on Thursday, a breakfast on Thursday, Friday or Saturday, or a luncheon on Friday, the local committee will help make these arrangements. Please write to Miss Mary C. Rogers, 307 Prospect Street, Westfield, New Jersey.

Location of Meeting Rooms

The Ambassador is headquarters for the

convention. All rooms and parlors used for the meetings and discussions are on the Lounge Floor or the First Floor of this hotel.

Mail and Telegrams

Mail and telegrams for those attending the convention should be addressed in care of The National Council of Teachers of Mathematics, The Ambassador, Atlantic City, New Jersey. Mail may be obtained at the registration desk in the Foyer on the Lounge Floor.

Refunds on Reservations

No ticket refunds will be made later than three hours preceding the function for which reservations were made.

Certificate of Attendance

If you care to take back to your school authorities a statement certifying your attendance at the convention, make request for it at the Registration Desk on Saturday morning.

PROGRAM COMMITTEE

John R. Mayor, Madison, Wisconsin, Chairman; Agnes Herbert, Baltimore, Maryland; Irene Sauble, Detroit, Michigan; Marie S. Wilcox, Indianapolis, Indiana; James H. Zant, Stillwater, Oklahoma.

LOCAL COMMITTEES

(Note: Members of committees are from the New Jersey schools unless otherwise indicated.)

General Chairman

Mary C. Rogers, Roosevelt Junior High School, Westfield.

Advisory Committee

Chairman—Carl N. Shuster, State Teachers College, Trenton, President of Association of Mathematics Teachers of N. J.; Amanda Loughren, Board of Education, City Hall, Elizabeth; Virgil S. Mallory, State Teachers College, Montclair; Albert E. Meder, Jr., Dean of Rutgers University, New Brunswick; Madeline D. Messner, Abraham Clark High School, Roselle; John K. Reckzeh, State Teachers College, Jersey City.

Convention Secretary

Gladys W. Estabrook, Cranford High School, Cranford.

Convention Treasurer and Chairman of Finance Committee

Hubert B. Risinger, Davey Junior High School, East Orange.

Additional Members of Finance Committee

Harold A. Fellows, East Orange; Dorothy Molinari, Atlantic City.

Publicity Committee

Chairman—Robert E. Luce, New Brunswick; Helen Clarke, Hackensack; Robert M. Coulter, Elizabeth; William Hausdoerffer, Trenton; Bernard Lyons, Elizabeth; John K. Reckzeh, Jersey City; Sister Anna Concillio, Convent Station; William S. Sterner, New Brunswick; Robert M. Walter, New Brunswick; Mary Ann Woodard, South Orange; Jackson B. Adkins, Exeter, New Hampshire; Ida May Bernhard, Austin, Texas; Eleanor V. Douglas, Washington, D. C.; Howard F. Fehr, New York City; John J. Kinsella, New York City; Rose Klein, New York City; Elizabeth Sibley, New York City; William A. Gager, Gainesville, Florida; Agnes Herbert, Baltimore, Maryland; Donovan A. Johnson, Minneapolis, Minnesota; Catherine A. V. Lyons, Pittsburgh, Pennsylvania; Alice M. Reeve, Rockville Centre, New York; John F. Schacht, Columbus, Ohio; Gladys Schuder Charlottesville, Virginia; Arthur A. Le Pori, East Orange.

Coordinator of Social Activities

Madeline D. Messner, Roselle, New Jersey.

Hospitality Committee

Chairman—Edith Day, Teaneck; Josephine Emerson, Summit; Louise Fisher Teaneck; Amada Loughren, Elizabeth; Virgil S. Mallory, Montclair; Frank J;

McMackin, Jersey City; George McMeen. Newark; Albert E. Meder, Jr., New Brunswick; Francyl Rickenbrode, New Brunswick; Sister Marie Isabel, North Arlington; Sister Miriam Mercedes, North Arlington; Millicent Snaith, Elizabeth; Emory P. Starke, New Brunswick; Inez E. Teague, Paterson; Adah Wade, Elizabeth; William Betz, Rochester, New York; H. W. Charlesworth, Denver, Colorado; H. C. Christofferson, Oxford, Ohio; Howard F. Fehr, New York; John J. Kinsella, New York; Mary A. Potter, Racine, Wisconsin; Elaine Rapp, Oceanside, New York; William D. Reeve, New York; Veryl Schult, Washington, D. C.; Edith Woolsey, Minneapolis, Minnesota.

Banquet Committee

Chairman—Euphemia Gibson, Kearny; Margaret Ault, Elizabeth; Eleanore Brown, Jersey City; Isadore Chertoff, Bayonne; J. Dwight Daugherty, Paterson; Foster E. Grossnickle, Jersey City; Harold Hamilton, Highland Park; Elaine Heinzelmann, Ridgewood; Ralph C. Miller, Ridgewood; Frances Nadel, Jersey City; Adelaide Pratt, Kearny; Laura Schefter, Newark.

Luncheon Committee

Chairman—Margaret Aeschbach, South Orange; Howard Ayers, Newark; Margaret Banta, Butler; Clara Collins, South Orange; Florence Darrah, Atlantic City; Susan Eveleigh, Atlantic City; Charlotte E. Godfrey, Passaic; Samuel M. Gordon, New Brunswick; Katherine Hughes, Elizabeth; Myrtle Hughes, Highland Park; May Jasper, Atlantic City; Helen Kaplan, Newark; Belle Kearney, Nutley; Roy Lockhart, Madison; Burnham Paige, South Orange; Dorothy Roberts, Scotch Plains; Mollie Updike, Princeton; Sol Whitman, New Brunswick.

New Jersey Reception

Chairman—Margaret M. Dunn, Bloomfield; Myretta F. Brinker, Collingswood; Philip Clamurro, Newark; Paul C. Clif-

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ADVANCE REGISTRATION AND RESERVATION FORM

Please fill out completely and mail with remittance to MISS GLADYS W. ESTABROOK, 42 W. Holly St., Cranford, N. J. before March 25, 1953. Make check or money order payable to convention treasurer, Hubert Risinger.

Mr., Miss, Mrs. ____

Last Name First Name Initial

Address ____

Street and Number City Zone State

Registering as: Member of NCTM ________ Member of MAA ________ Non-Member ________ Elementary Teacher ________ Student ________ Exhibitor ________

Check your field of teaching interest: Elementary ________ J.H.S. ________ S.H.S. ________ Teacher Education ________ Supervision ________ Other ________

Registration Fee $0.50 or $1.50 ________ Amount ________

New Jersey Shore Dinner *Number @ $4.60 ________ Amount ________

Luncheon-Saturday *Number @ $3.20 ________ Amount ________

* **Price includes gratuities.**

Total Amount Enclosed $________

Continuity Group: ________ Leader ________

Mathematics Laboratories (Register for one only) Thursday Elementary ________ Thursday J.H.S. ________ Thursday S.H.S. ________ Friday Secondary ________

Enrollments in Continuity Groups and in Mathematics Laboratories will be made as registrations come in. The number enrolled may be limited, especially in the Laboratories.

ford, Montclair; Lina Dunne, Upper Montclair; Regina H. Garb, Bloomfield; Mary Etta Higgins, Bloomfield; Verna Jovan, Bloomfield; Ellis R. Ott, New Brunswick; Sister M. Mercidita, Montclair; George C. Vedova, Newark; E. Paul Bridenbaugh, East Orange.

Information

Chairman—Mary Ferguson, Atlantic City; Frances McDonald, Atlantic City; Chester Olinger, Collingswood; Bertha Palmer, Glen Rock; Margaret VanDeusen, Mountain Lakes; Betty Wright, Morristown; Agnes Zofay, Union; Florence F. McHale, Atlantic City; Sara Stump, Swedesboro.

Ushers

Co-Chairmen—Lloyd E. Spangler, Atlantic City and T. R. Humphreys, Montclair. Members of the Committee will be mathematics students from the New Jersey State Teachers Colleges, and members of the Mathematics Club at the Atlantic City High School.

General Chairman of Exhibits

Ernest R. Ranucci, Newark

Commercial Exhibits Committee

Max A. Sobel, Newark

School Exhibits Committee

Chairman—Ernest R. Ranucci, Newark; Bernard Lyons, Elizabeth; Lentz D. Gold, Atlantic City; Amelia Richardson, McKeesport, Pennsylvania; Christina Little, Boston, Massachusetts.

Teaching Aids Committee

Chairman—Frances Gooen, Newark; Eleanor Wilsey, Newark.

Registration Committee

Chairman—Lloyd E. Spangler, Atlantic City; Robert T. Hughes, Atlantic City; Herbert Freed, Atlantic City; James A. Bewley, Atlantic City; Mary Dukes, Atlantic City; Esther Werntz, Atlantic City. (Other teachers from the Atlantic City schools will serve on this committee as needed.)

General Chairman of Arrangements in Atlantic City

May J. Kelly, Atlantic City.

Printing and Signs Committee

Chairman—Sylvia Good, Atlantic City; Mary A. Walsh, Atlantic City; Hessie Krechmer, Atlantic City.

Secretaries Committee

Chairman—Marion W. Fox, Atlantic City; Marion Kiehl, Atlantic City; Dorothy Whelan, Atlantic City; Robert Shaw, Atlantic City; Griselda Skidmore, Atlantic City; Raymond Hurter, Atlantic City.

Supplies and Equipment Committee

Chairman—Wilma L. Nelson, Atlantic City; Teresa Granata, Atlantic City; Russell Lamkin, Atlantic City.

School Visitation Committee

Chairman—Eleanor Helfrich, Atlantic City; Charles R. Hollenbach, Atlantic City; Samuel A. Gillingham, Atlantic City; Harry M. Fagan, Atlantic City; Mabel C. Castle, Atlantic City; John Craven, Atlantic City; T. Montgomery Gregory, Atlantic City; Elizabeth D. Vance, Atlantic City; W. Hibberd Smith, Atlantic City; Aubrey L. Hoxter, Atlantic City; Emily H. Surtees, Atlantic City; Helen R. Rouse, Atlantic City.

Please Note: In making hotel reservations please include choice and number of rooms desired as well as times and dates of arrival and departure. Send your requests directly to the Ambassador Hotel, Atlantic City, New Jersey.